Summary of Observation in Cosmology

1. Cosmic Expansion

Hubble's law（1929）

$$v = H_0 d$$

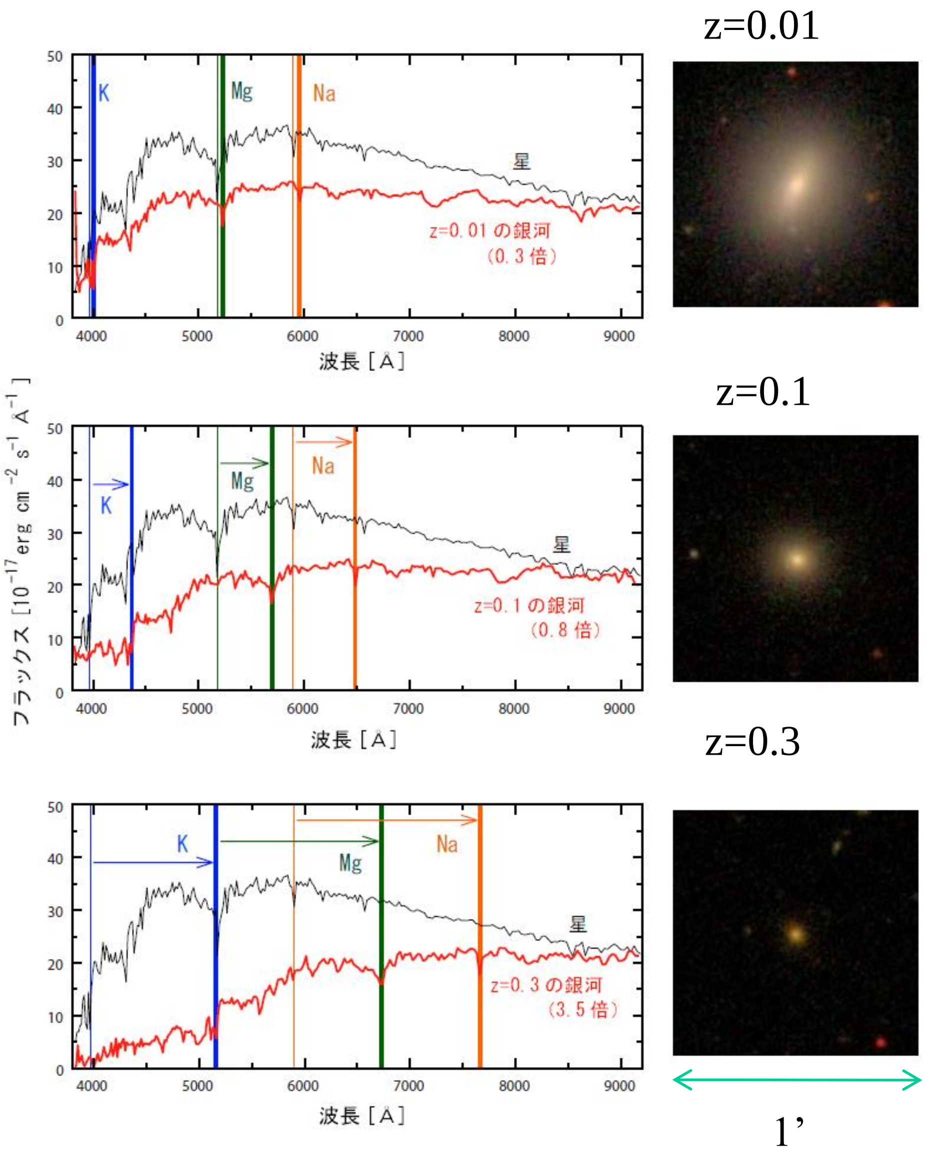
H_0： Present Hubble parameter

$$h \equiv H_0 /(100\ \mathrm{km\ s^{-1} Mpc^{-1}})$$
$$\approx 0.7$$

For small z<<1

$$z = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}} \approx \frac{v}{c} \qquad v << c$$

$$d = \frac{cz}{H_0}$$



by K. Yawata、NOAJ

Data by Hubble

1929 by Hubble

$$m(z)-M=5\log_{10}\left(\frac{d_L(z)}{10\,pc}\right)$$

Cepheid variable as standard candle

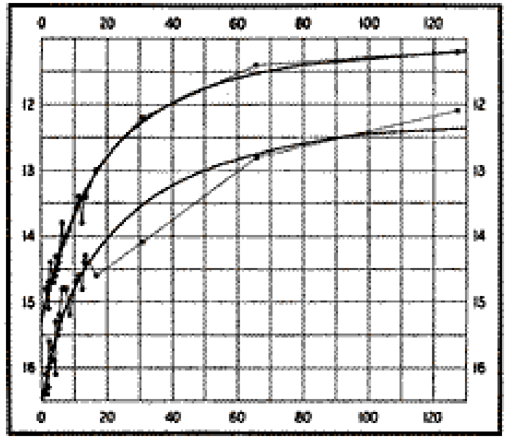


FIG. 1.

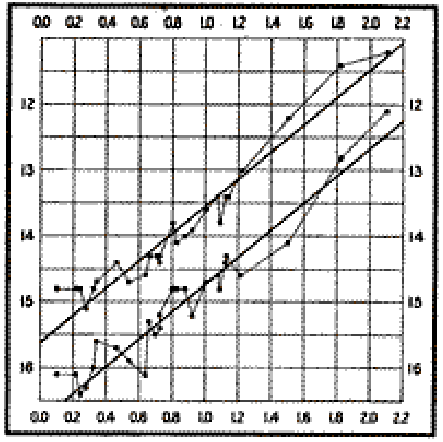
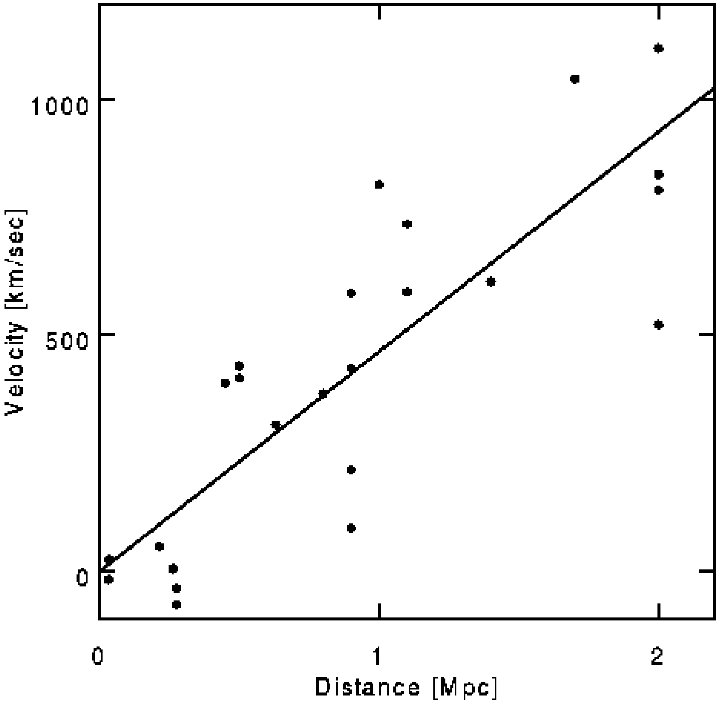


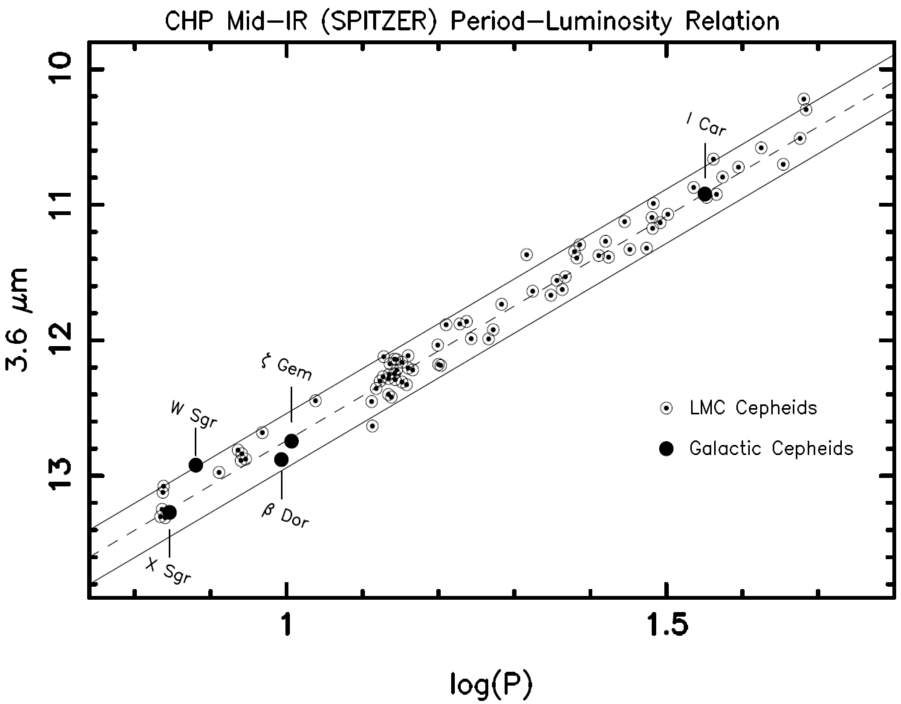
FIG. 2.

Data by Leavitt(1912)



$$H_0 \approx 530\ \mathrm{km/s/Mpc}$$

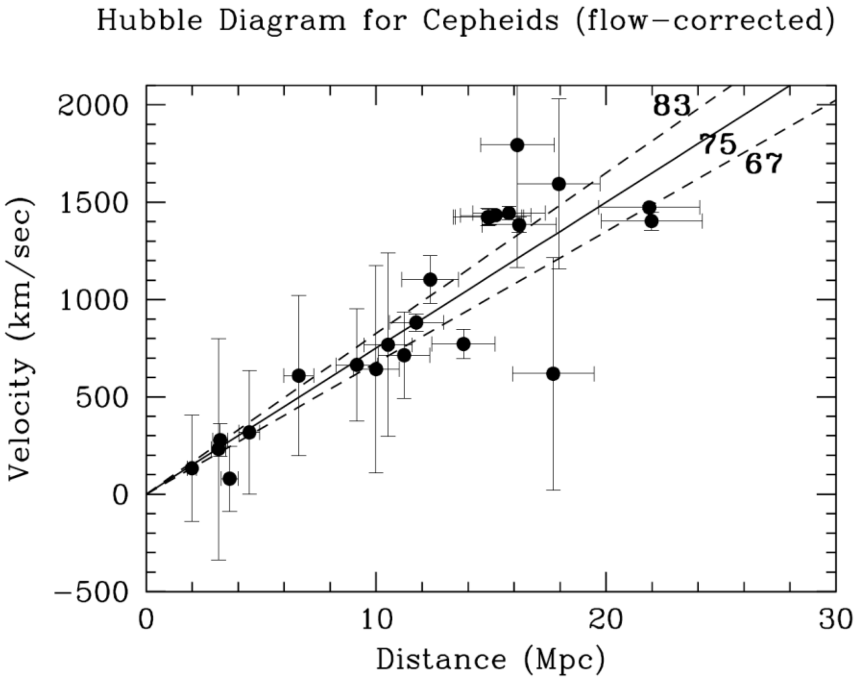
Recent observation



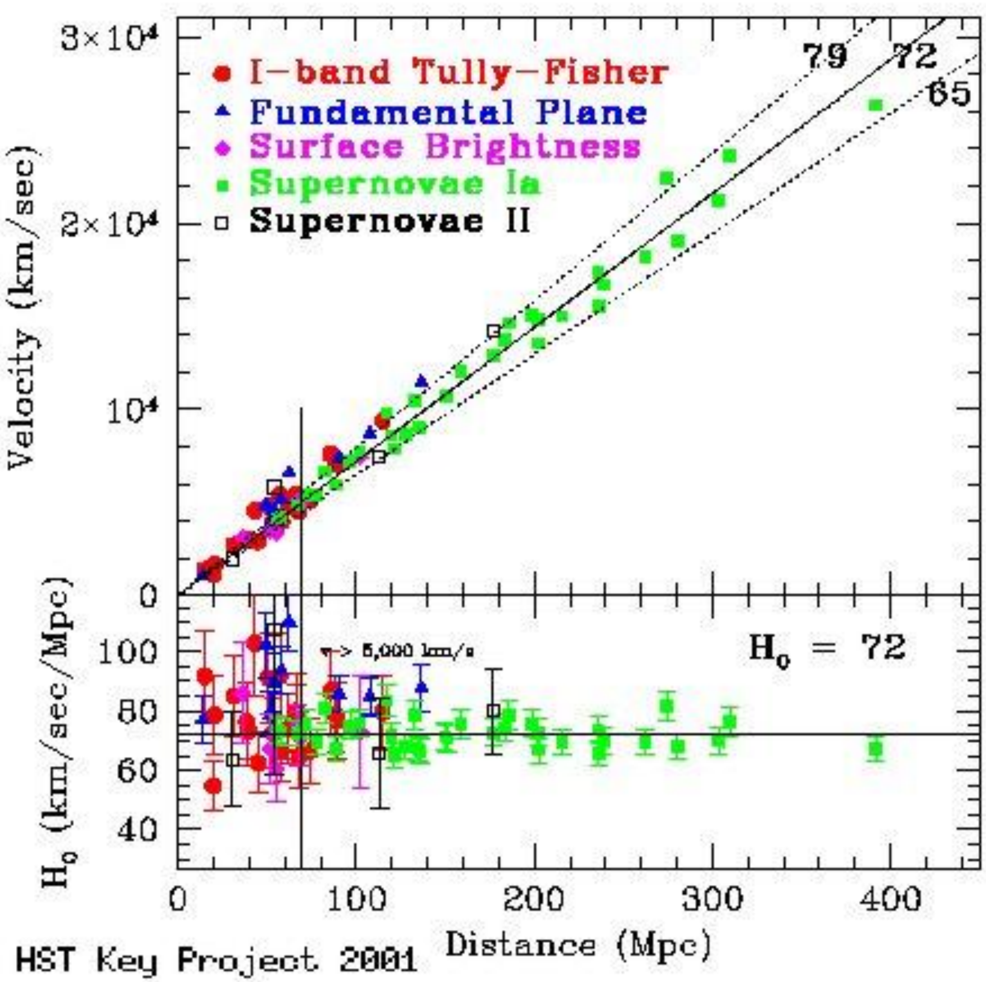
Period-luminosity of Cepheid variables observed relation Spitzer Infrared Space Telescope

Freedman et al. AJ, 2011

Hubble law by Cepheid discovered by HST



Various Standard Candle



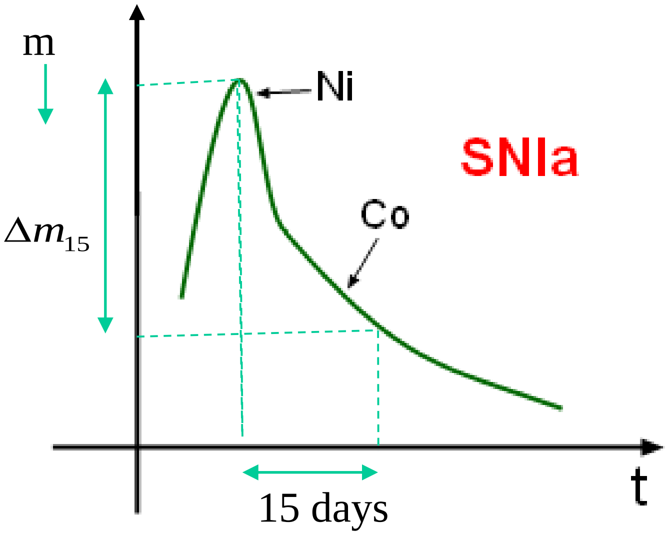
HST Key Project 2001

Disk galaxies

Tully-Fisher rel. $L \propto v^4$

Type Ia SN

$$M_B \cong 0.8(\Delta m_{15} - 1.1) - 19.5$$



⇨ $H_0 = 72 \pm 8$ km/s/Mpc

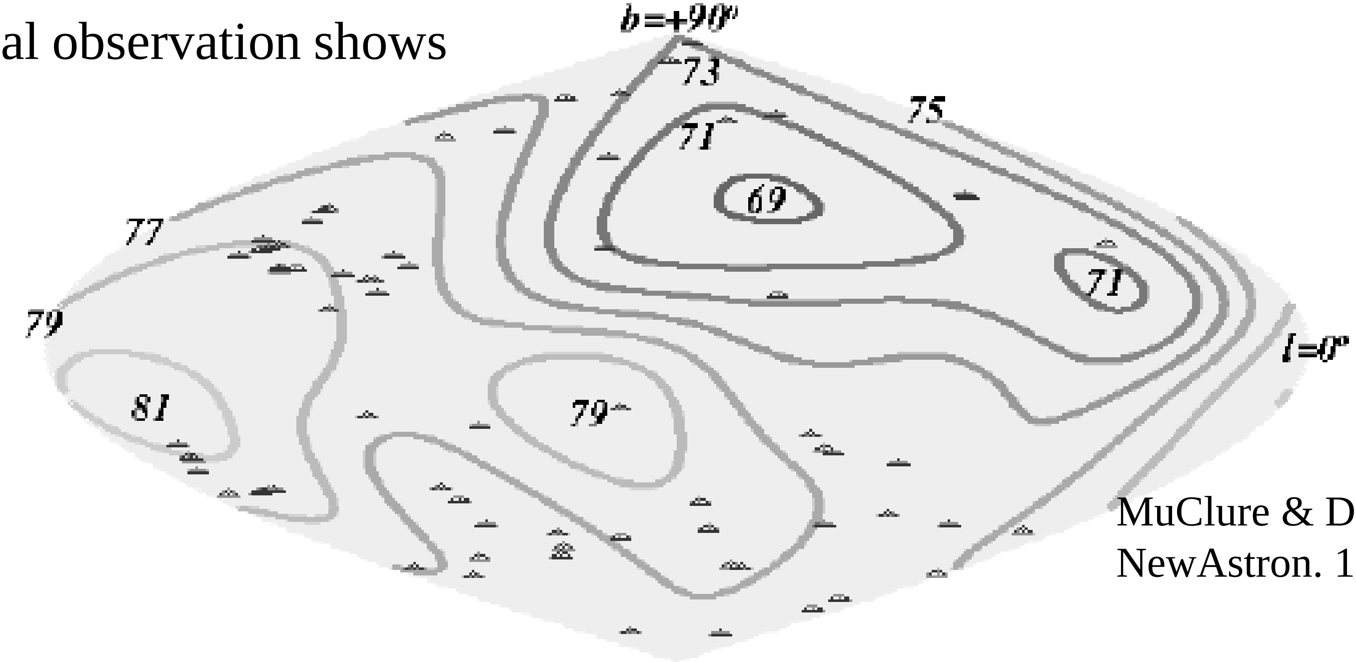
Light curve of Ia SN

Is Universe expanding isotropically？

Linear relation between vectors should be

$$\vec{v} = H_0 \vec{d} \Rightarrow v^i = (H_0)^i_j \, d^j$$

Actual observation shows



MuClure & Dyer
NewAstron. 12,2007)

Directional dependence is the result of local gravitational force by neibhoring galaxies

$$\vec{v} = H_0 \vec{d} + \vec{v}_{pec}$$

Hubble flow

Peculiar velocity(特異速度)

Importance of Hubble parameter

It gives atypical scale of the universe

$$H_0^{-1} \approx 3.1\times10^{17}\, h^{-1}\mathrm{sec} \approx 9.8\times10^{9}\, h^{-1}\mathrm{yr}$$ Hubble time

$$h \equiv H_0 /(100\ \mathrm{km\ s^{-1}Mpc^{-1}})$$

$$cH_0^{-1} \approx 3{,}000\, h^{-1}\mathrm{Mpc}$$ Hubble distance

$$\rho_{cr,0} \equiv \frac{3H_0^2}{8\pi G} \approx 1.88\times10^{-29} h^2\ \mathrm{g/cm^3}$$ Critical density

$$\Omega_{X,0} \equiv \frac{\rho_{X,0}}{\rho_{cr,0}}$$ Density parameter

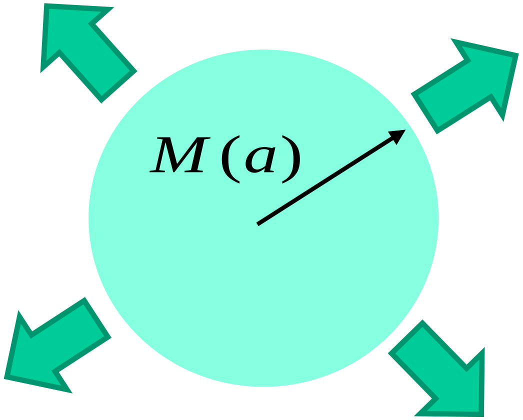
2つの銀河の間隔を $d(t) = a(t)\ell_0$ (ℓ_0 は一定）と書くと

$$v = \dot{d} = \dot{a}\ell_0 = \frac{\dot{a}}{a}(a\ell_0) = Hd \Rightarrow H = \frac{\dot{a}}{a}$$

宇宙の膨張を表す関数 $a(t)$ をスケール因子（scale factor）という

ハッブルパラメータ H は時間の関数で、現在の値を H_0 と書く

Dynamics of an expanding universe



Total gravitational energy of universe

$$\frac{1}{2}\dot{a}^2 - \frac{GM(a)}{a} = E$$

Kinetic energy Potential energy

Let ρ be the energy density, then the total energy contained in a sphere with radius a

$$M(a) = \frac{4\pi}{3}a^3\rho :$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} \qquad K = -2E \qquad \text{Eq.(A)}$$

Thus the flat universe(K=0) has 0 gravitational energy

Equation for acceleration

For the moment we assume that Universe contains 2kinds of energy

- Non-relativistic matter (baryon+Dark matter) : $\rho_m \propto \frac{1}{a^3}$
- Radiation (massless particle) : $\rho_r \propto \frac{1}{a^4}$

Both evolution can be described by the following

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + P(\rho)\right) = 0 \qquad \text{Eq.(B)}$$

since $P_m \approx 0,\ P_r = \frac{1}{3}\rho_r$

Combined Eq.(A) and (B), we have

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3P\right)$$

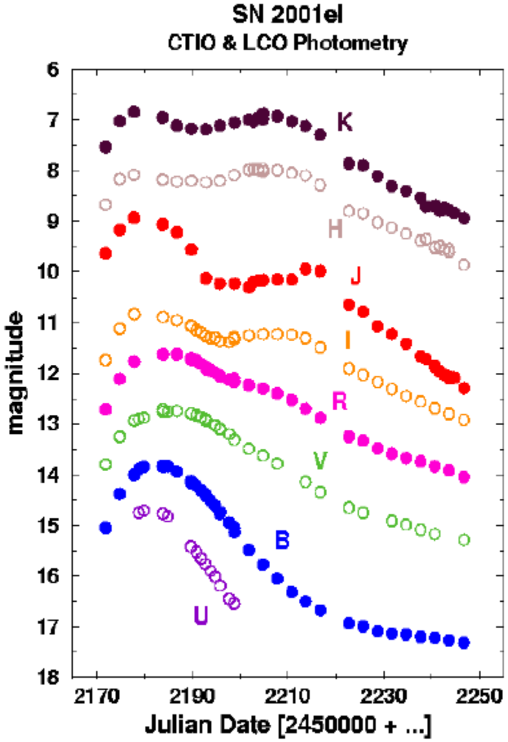
➡ $\ddot{a} > 0$ as far as $\rho + 3P > 0$

Now we have Supernpvae data

Observation of type Ia Supernovae

$$m(z) - M = 5\log_{10}\left(\frac{d_L(z,\Omega_m,\Omega_\Lambda)}{10\,pc}\right)$$

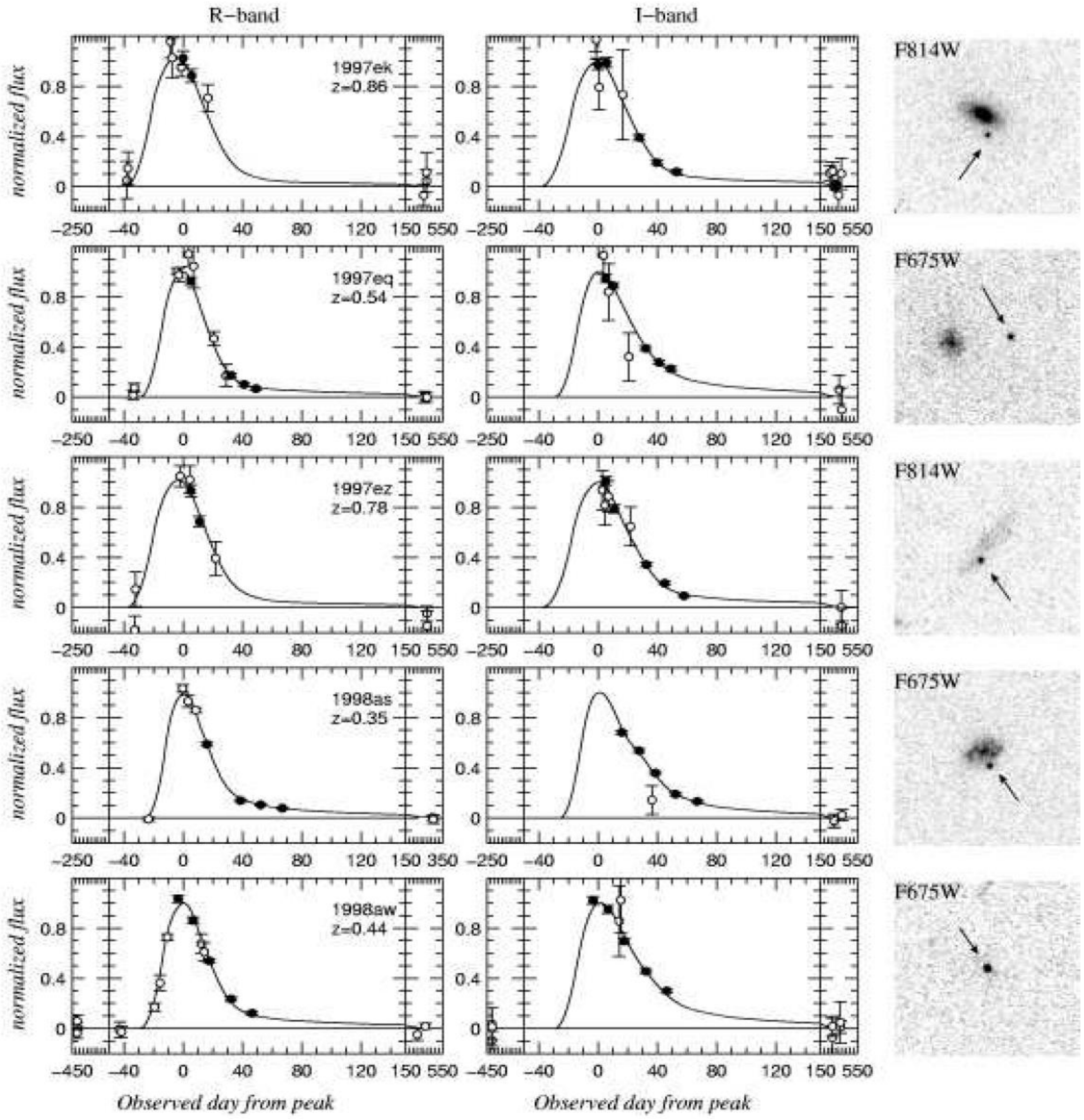
渦巻銀河 NGC4526 中で爆発した Ia 型超新星 SN1994d．図の左下には，もともと何も見えなかったが，突如銀河全体に匹敵するほど明るさで超新星が現れた．(出典：http://hubblesite.org/)．



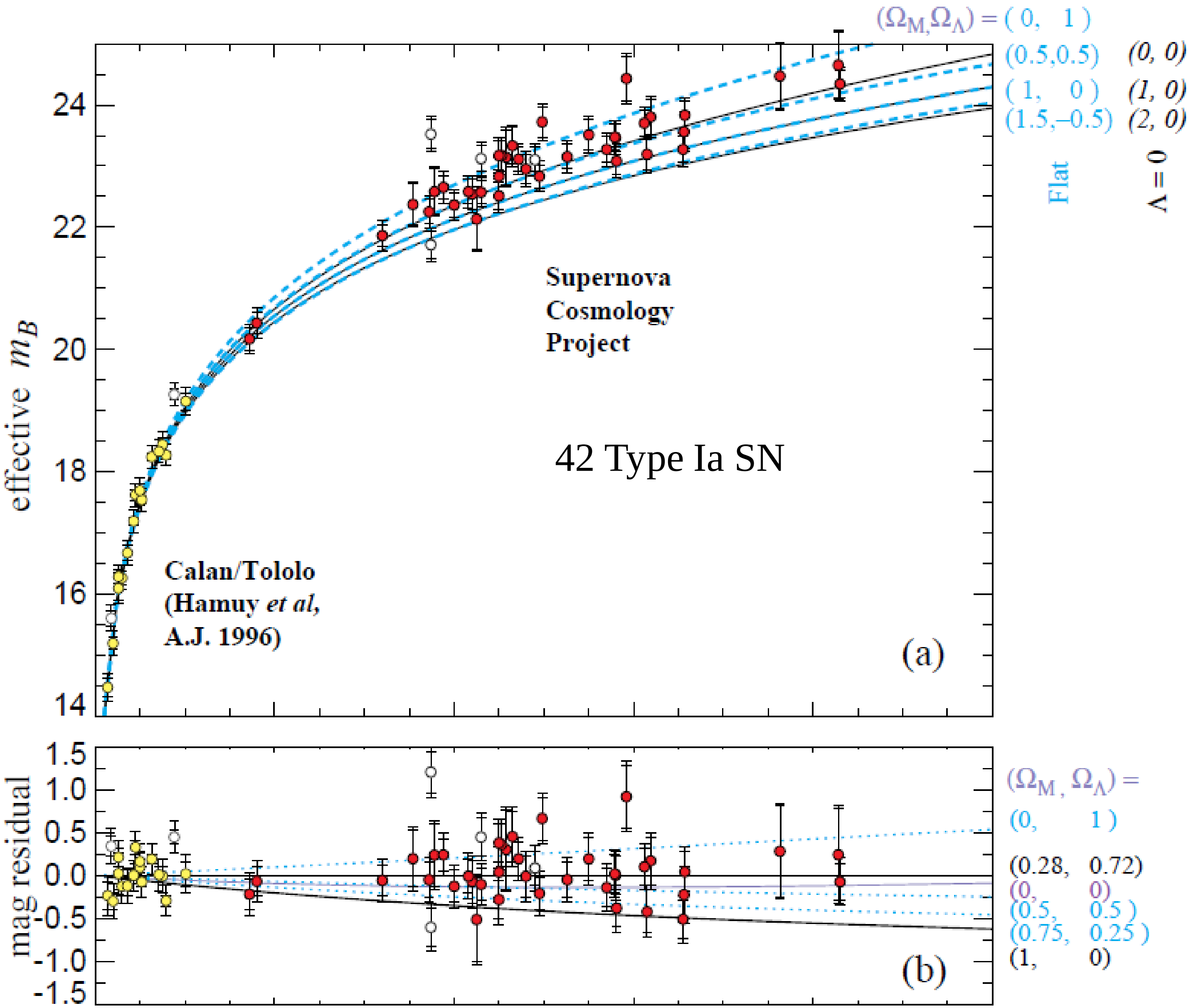
Ia 型超新星 SN2001el の光度曲線．縦軸は様々なバンドでの光度，横軸は日数を表す (Krisciunas et al. 2003, *AJ*, 125, 166).

Discovery of accelerated expansion

Type Ia super novae as a standard candle

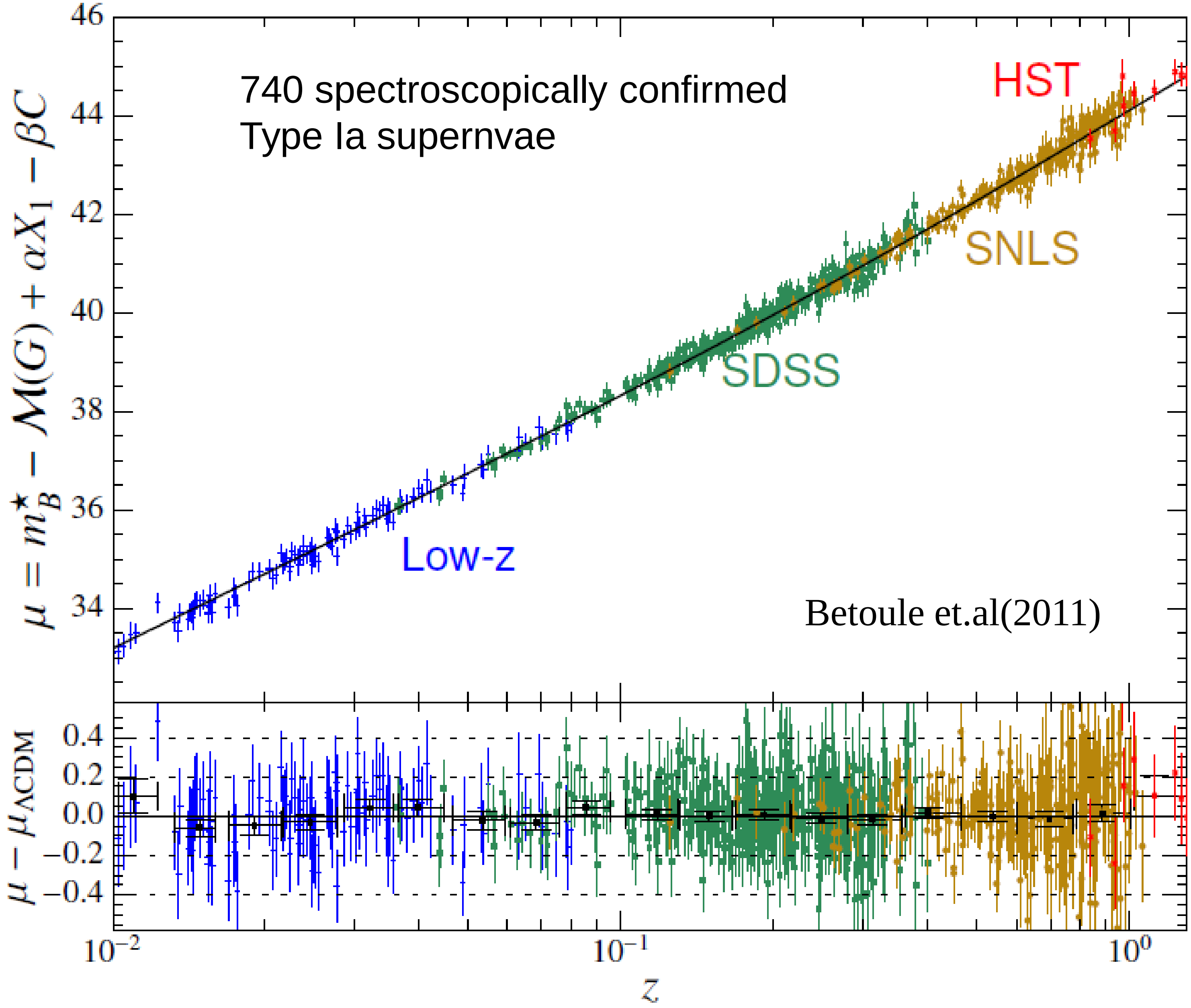


Knop et al. 2003



42 Type Ia SN

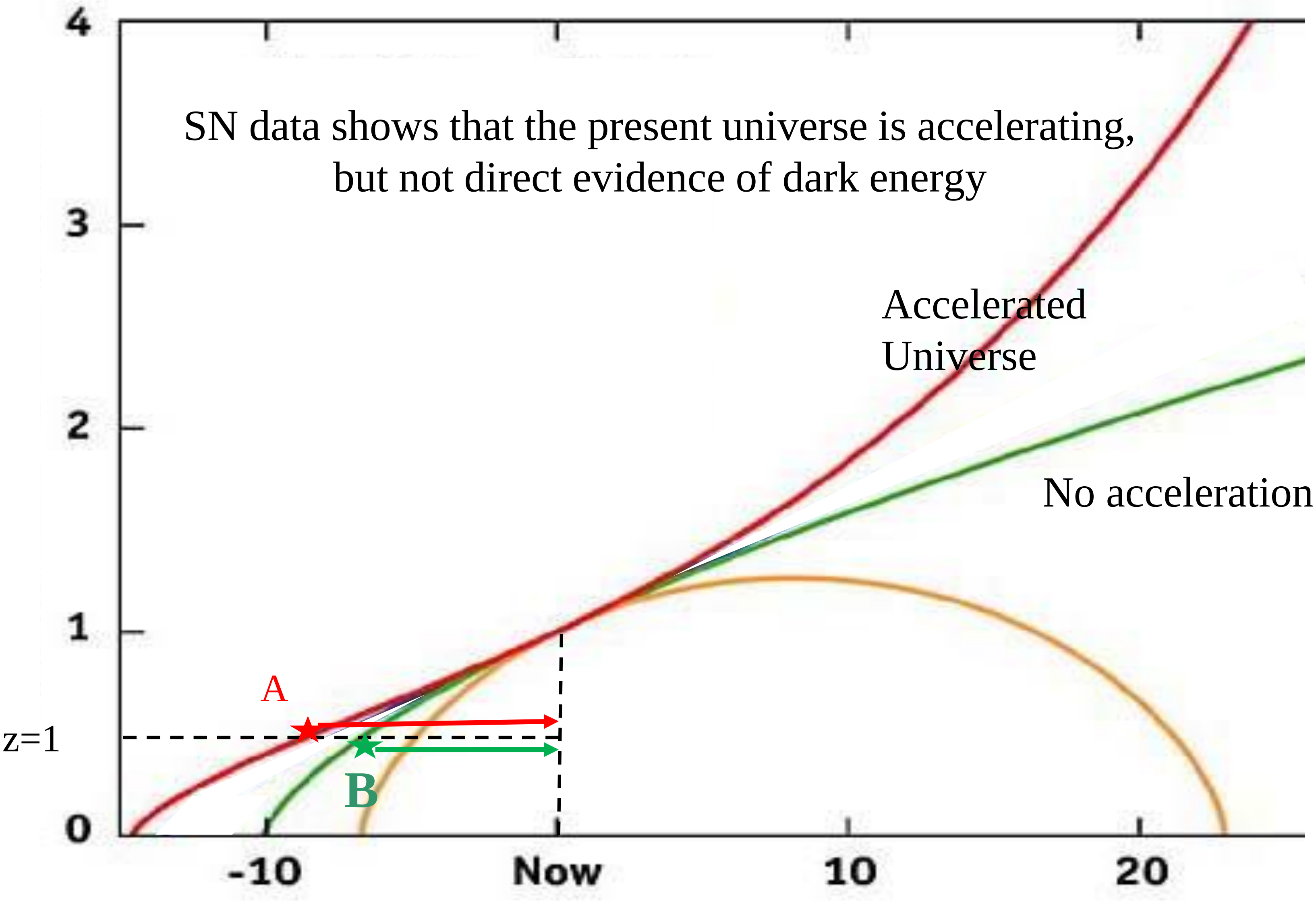
Perlmutter et al (1998)



740 spectroscopically confirmed
Type Ia supernvae

Betoule et.al(2011)

$\Omega_{m,0} = 0.295 \pm 0.035$ for a flat ΛCDM cosmology with H_0=70 km/s/Mpc



SN data shows that the present universe is accelerating,
but not direct evidence of dark energy

MAP990350

Dark energy

If Einstein’s gravity is collect over cosmological scale

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3P\right) > 0$$

means that there is a energy which satisfies

$$\rho + 3P < 0$$

This is called dark energy.

Cosmological constant is a special case of dark energy

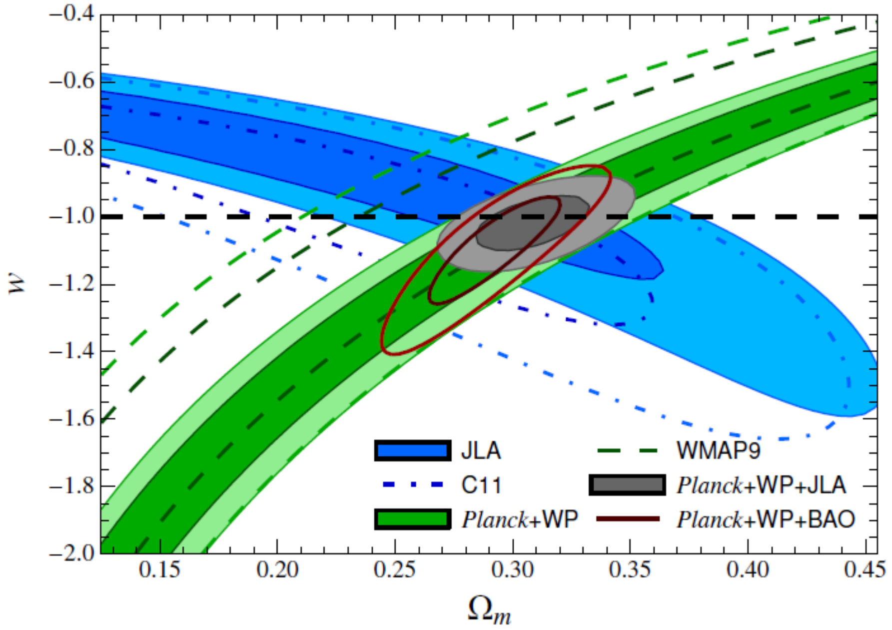
$$\rho_\Lambda = -P_\Lambda$$

In this case

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + P\right) = 0 \Rightarrow \dot{\rho}_\Lambda = 0$$

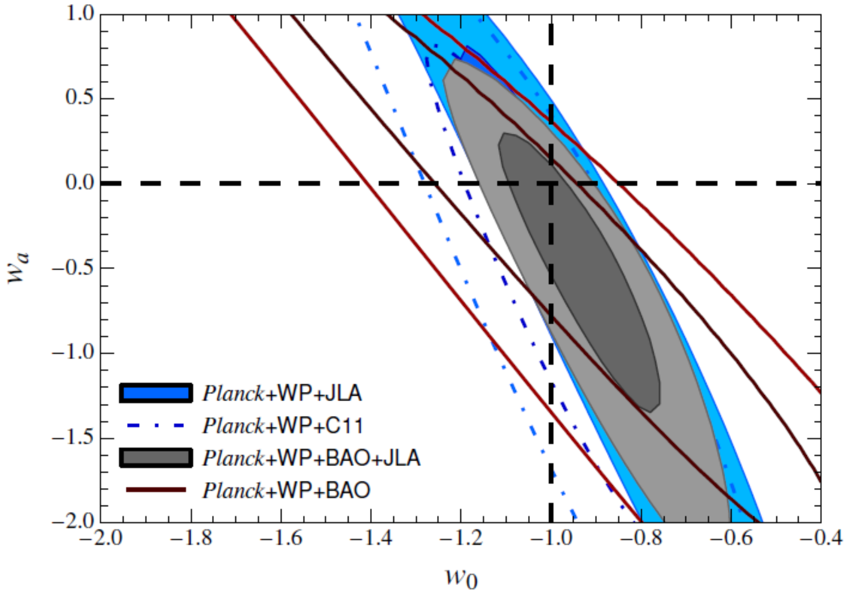
In general $P_{DE} = w(z)\rho_{ED}$ with $w < -1/3$

Example: $w(z) = w_0 + w_a(1-a) = w_0 + w_a \dfrac{z}{1+z}$



$$w_0 = -1.08 \pm 0.057$$

$$(w_a \equiv 0)$$

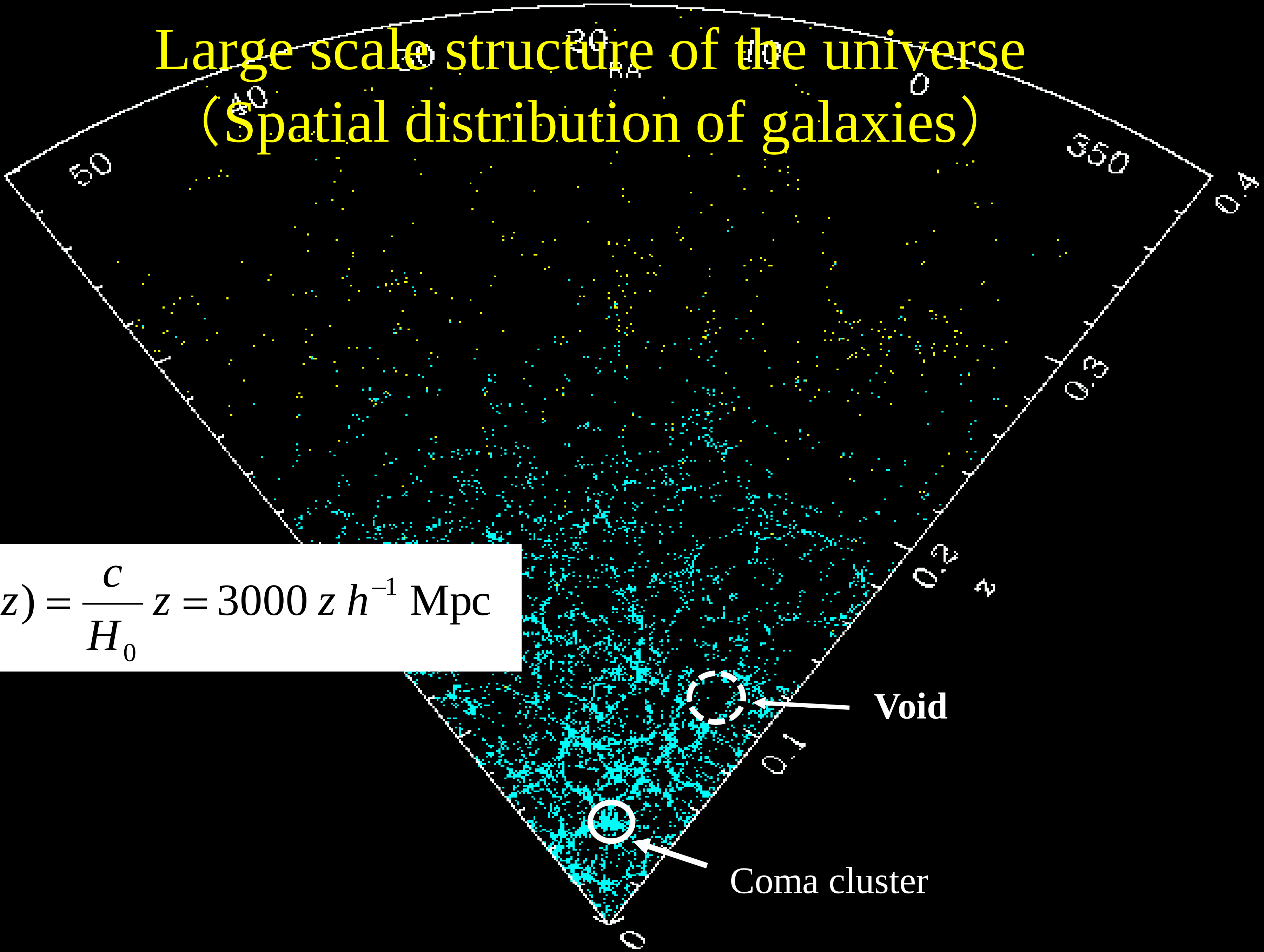


$$w_0 = -0.957 \pm 0.124$$

$$w_a = -0.336 \pm 0.552$$

2. Galaxy Survey

Large scale structure of the universe
（Spatial distribution of galaxies）



$$d(z) = \frac{c}{H_0} z = 3000\, z\, h^{-1}\ \mathrm{Mpc}$$

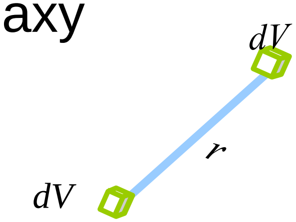
Void

Coma cluster

Description of LSS

2point correlation function

Probability to find another galaxy at distance from a galaxy



$$dP = \bar{n}_g{}^2 [1 + \underline{\xi_g(r)}] dV^2 \qquad \bar{n}_g \; dV \; << 1$$

What we observe is the number density of galaxy

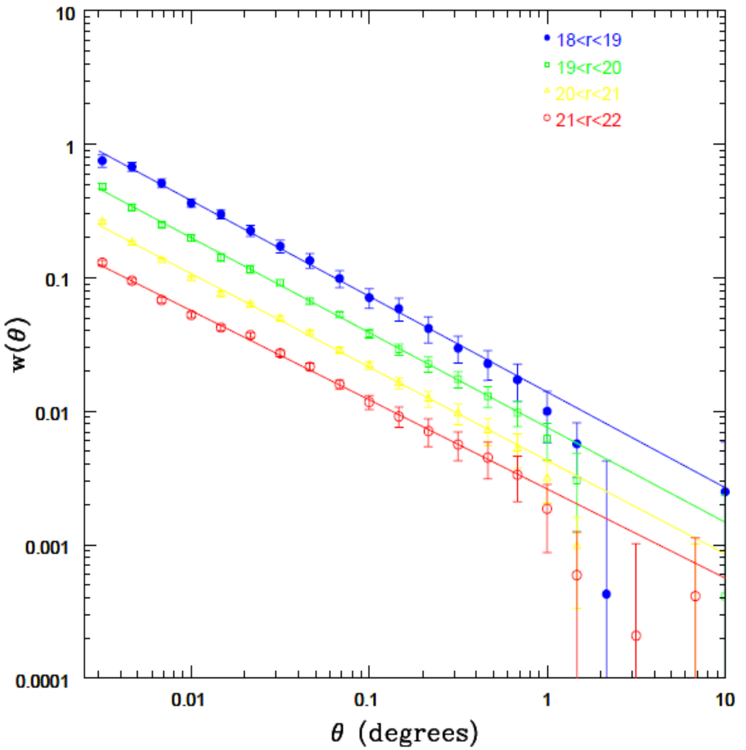
$$n_g(\vec{x}) = \bar{n}_g + \delta \; n_g(\vec{x})$$

$$\bar{n}_g{}^2 \xi_g(r) = \left\langle \delta n_g(\vec{x}) \delta n_g(\vec{y}) \right\rangle, \quad r \equiv |\vec{x} - \vec{y}|$$

Result from SDSS

$$\xi_g(r) = \left(\frac{r}{r_0}\right)^{-\gamma}, \; r_0 \simeq 5h^{-1}\mathrm{Mpc}, \; \gamma \simeq 1.8$$

$$2h^{-1}\mathrm{Mpc} \le r \le 30h^{-1}\mathrm{Mpc}$$



Connolly et al. 2002

Power Spectrum

Fourie transform of 2 pt. correlation function

$$P_g(k) \equiv \int d^3x\ e^{-i\vec{k}\cdot\vec{x}} \xi_g(r)$$

$$P_g(k) = \int d\Omega dr\ \ r^2\ e^{-ikr\cos\theta} \xi_g(t,r) = 4\pi \int_0^\infty dr\ r^2 \xi_g(r) \frac{\sin kr}{kr}$$

Here we assume that distribution of galaxy traces the distribution of dark matter(Light traces matter)

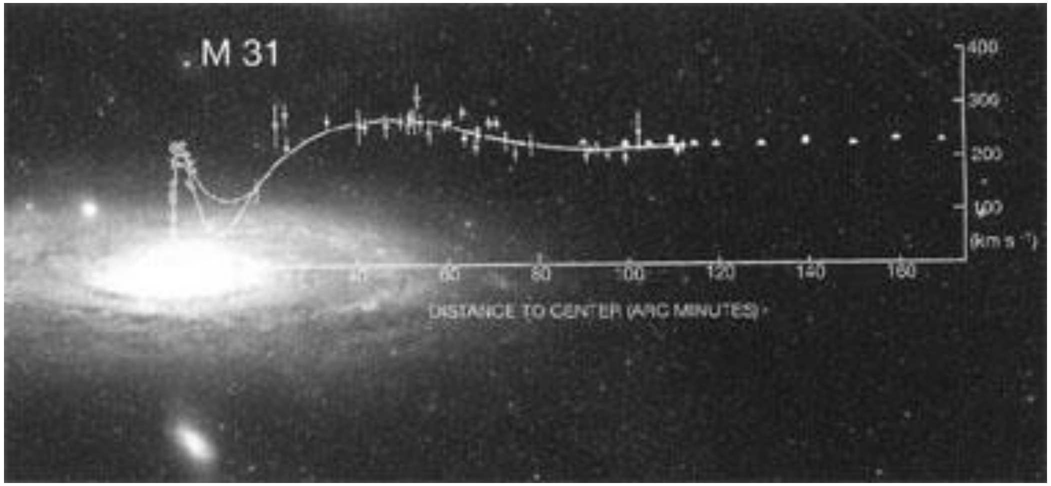
$$\rho_{\mathrm{DM}} = \bar{\rho}_{\mathrm{DM}}(1 + \delta_{DM})$$

$$\delta_{DM}(\vec{x}) = b\delta_g(\vec{x})$$

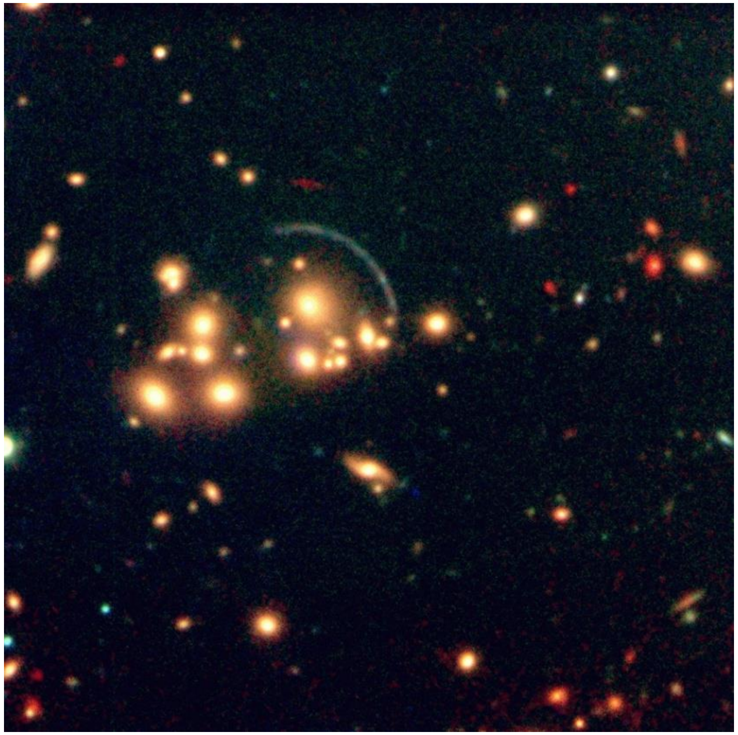
b : bias parameter

Evidence for the existence of Dark Matter1

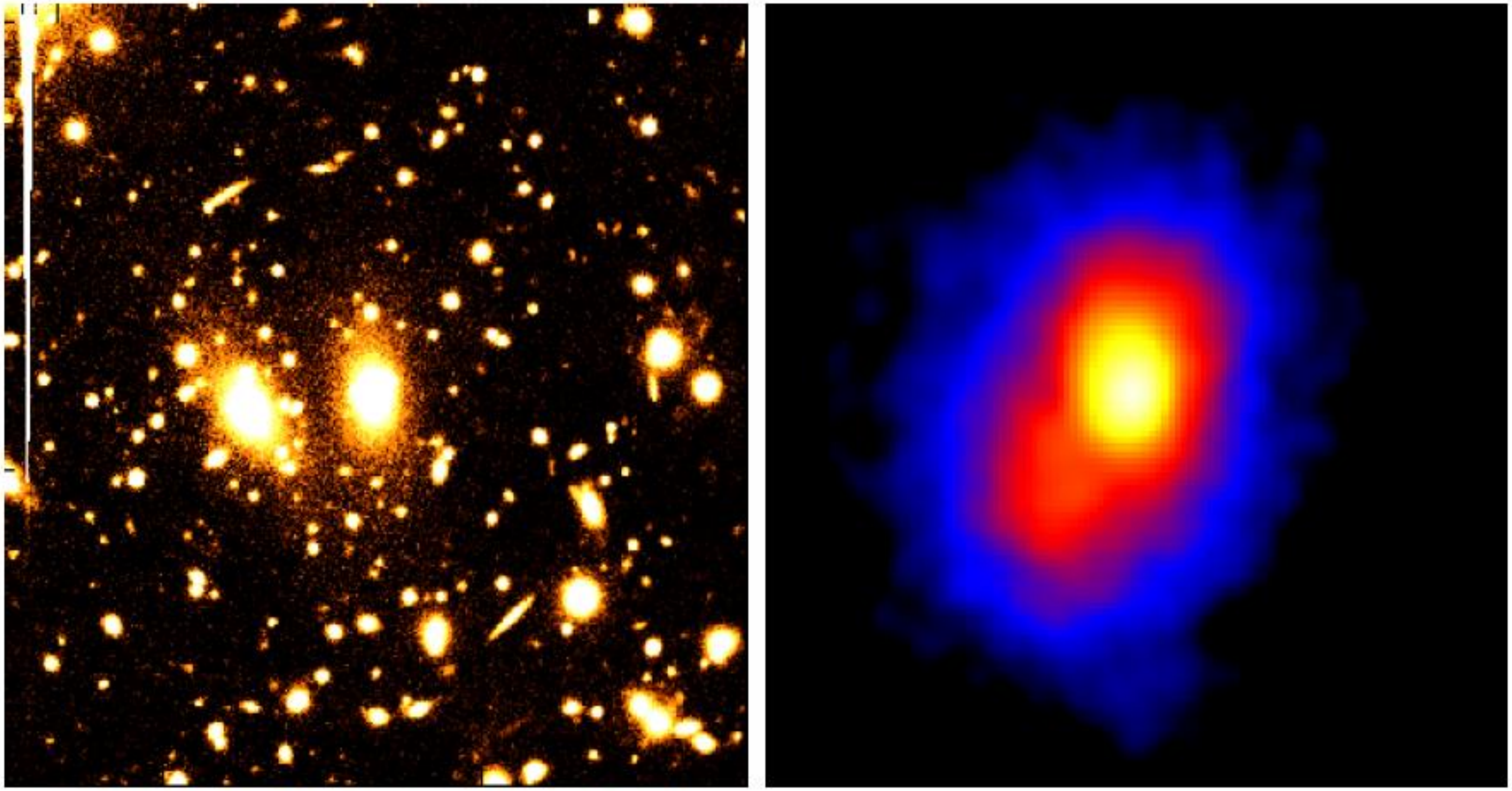
- Large velocity dispersion of member galaxies in cluster
- Flat rotation curve in spiral galaxies
- X rays from clusters, elliptical galaxies
- Gravitational lens



Flat rotation curve in M31

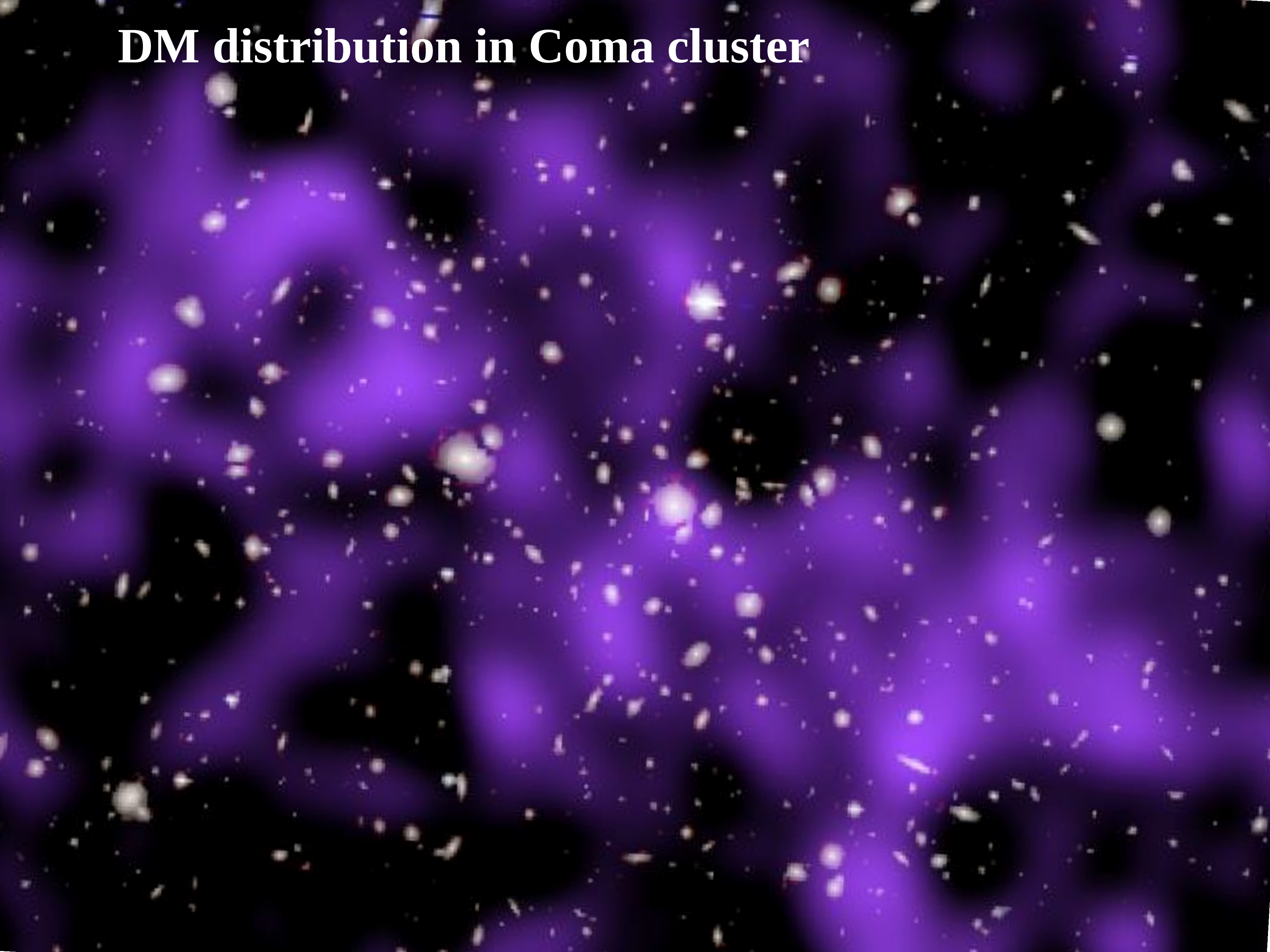


Giant arc observed in CL2244-04(z=0.3)



Optical and X ray images of galaxy cluster RX J1347.5-1145 .
Both are same scale about 600kpc long

DM distribution in Coma cluster



3types of Dark Matter

- Hot Dark Matter(HDM)

The velocity dispersion is of the order of speed of light at radiation-matter equality

Small scale structures are wiped away and large scale objects form first

Top-down scenario

- Cold Dark Matter(HDM)

The velocity dispersion is much less than the spped of light at

Radiation-matter equality

Bottom-up scenario

- Warm Dark Matter(WDM)

The prediction of SDM and its observation

Averaged density profile for relaxed CDM clumps is well approximated by a universal profile called NFW profile

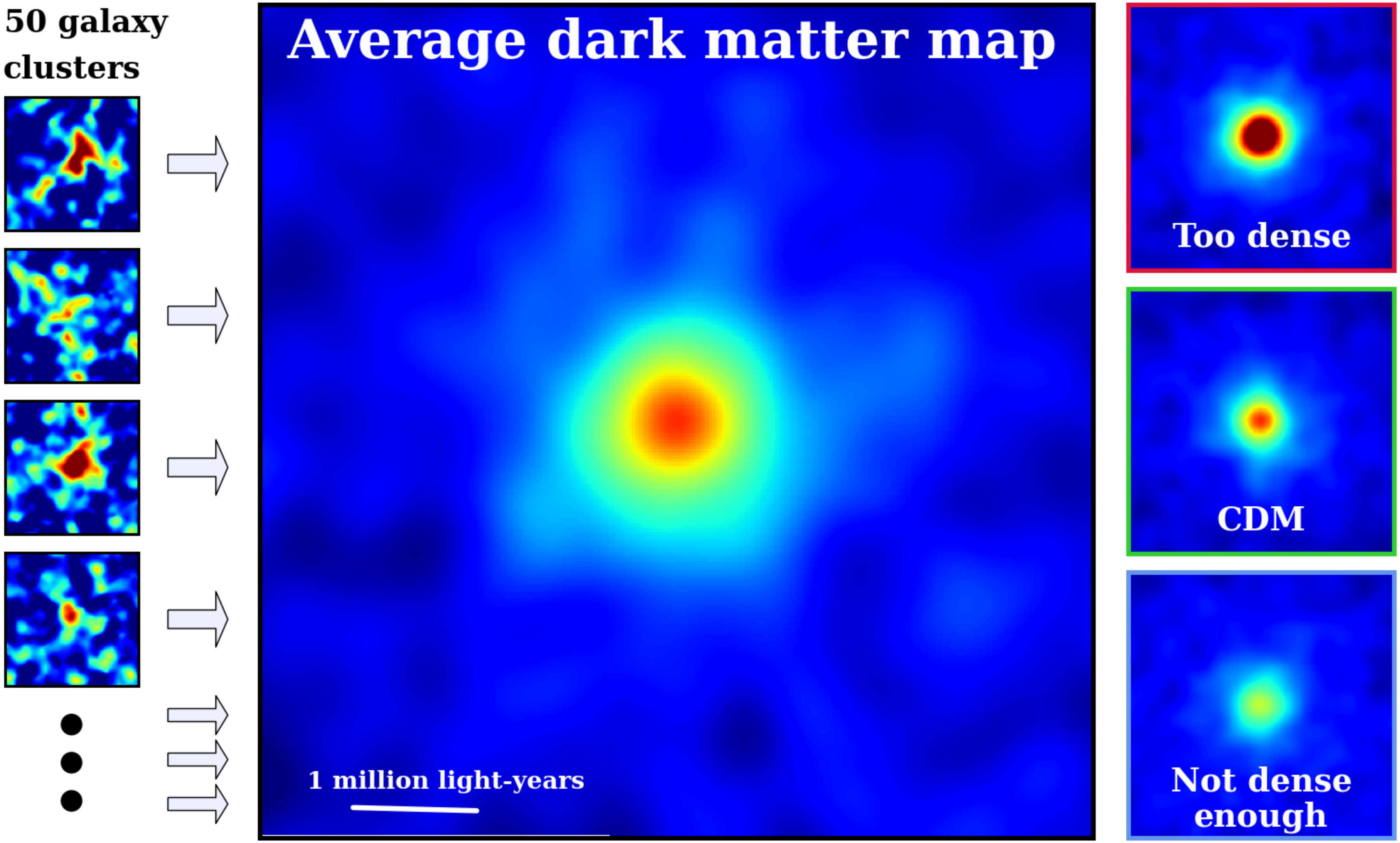
$$\rho(r) = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2}$$

The mass inside the radius r_Δ

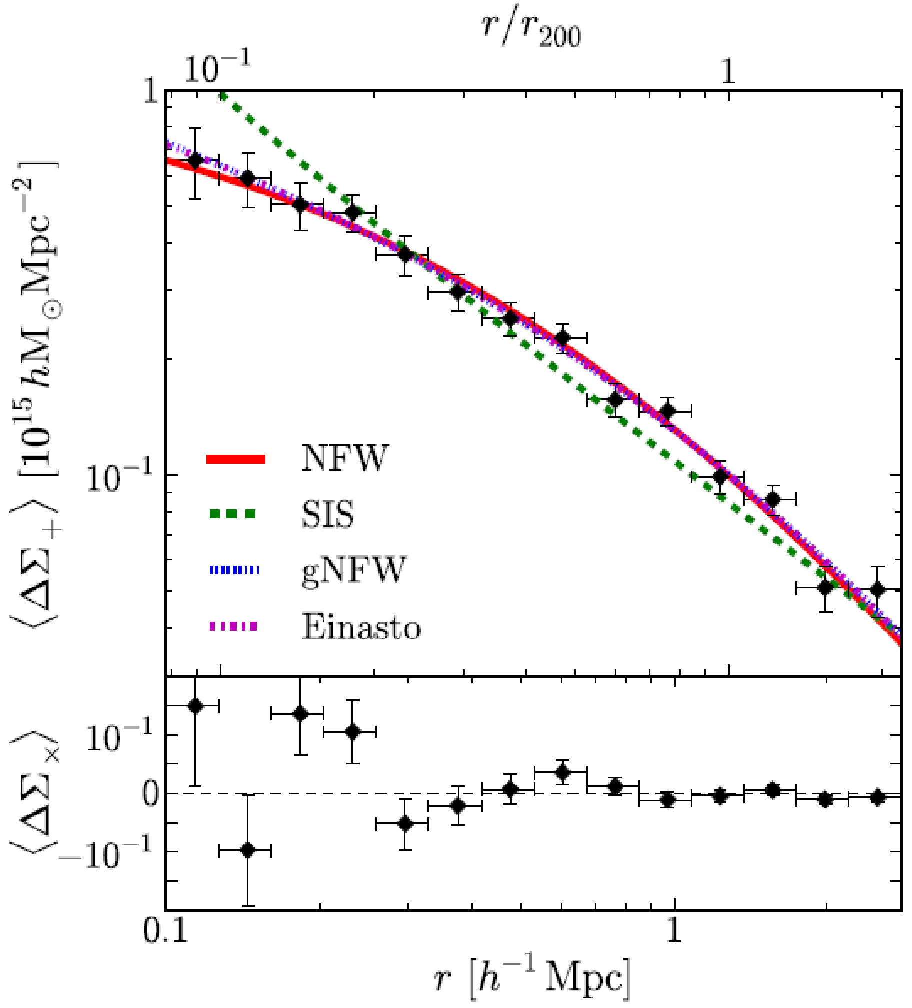
$$M_{\mathrm{NFW}}(r_\Delta) = \frac{4\pi\rho_s r_\Delta^3}{c_\Delta^3}\left[\ln(1 + c_\Delta) - \frac{c_\Delta}{1 + c_\Delta}\right]$$

where $c_\Delta \equiv \frac{r_\Delta}{r_s}$ is called the concentration.

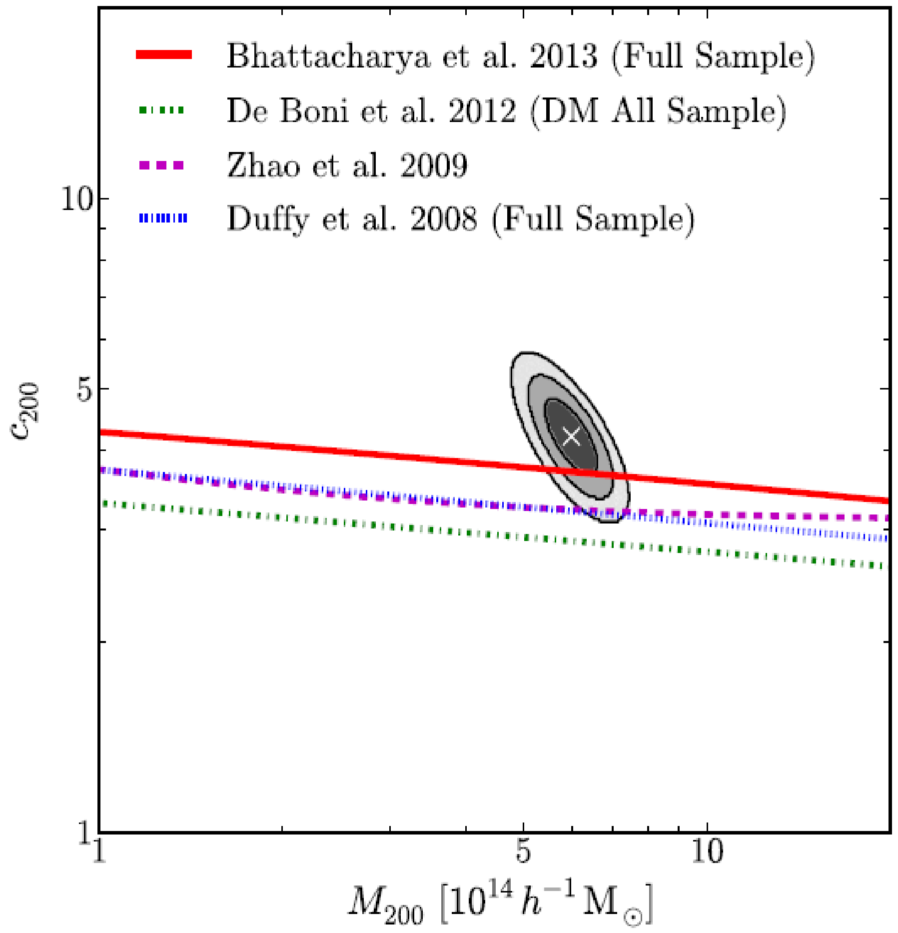
Weak Lensing Analysis for 50 clusters (0.15 <z<0.30) with Subaru



より詳しく



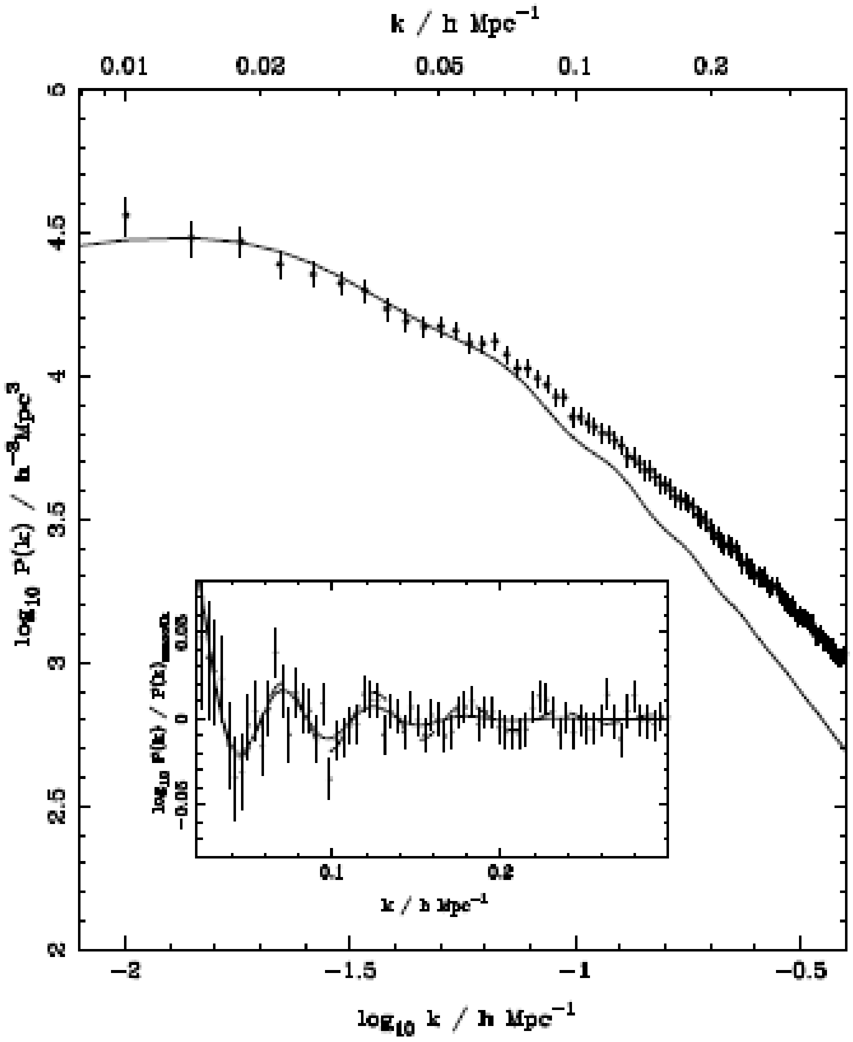
The averaged tangential shear profile obtaind from stacking 50 clusters with <z>-0.23



Mass and concentration of 50 clusters at <z_l>=0.23

Observed Galaxy Power Spectrum

THE SHAPE OF THE SDSS DR5 GALAXY POWER SPECTRUM



Percival et al.
2008

Redshift space distortion

Observed distance

$$d_{rss} \equiv \frac{1}{H_0}\left(cz + v_{pec,//}\right)$$

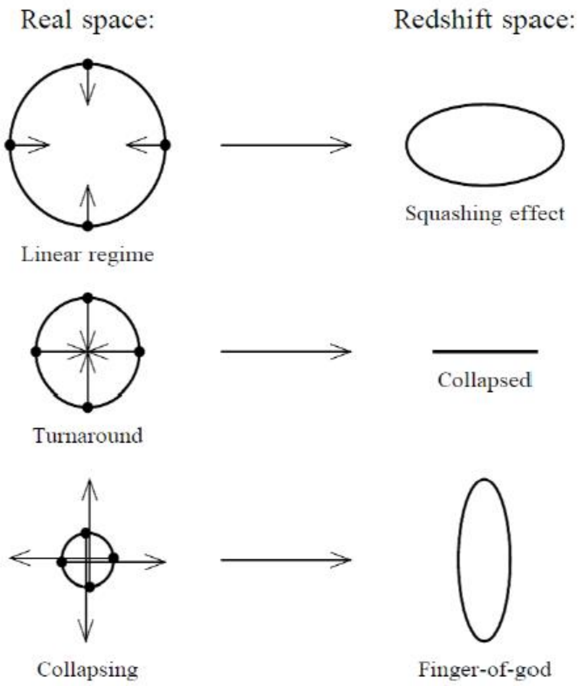
Since peculiar velocity is <1000 km/sec, galaxies less than ~15 Mpc from us is affected by this effect for $H_0 \sim 70\,\frac{km}{s}/Mpc$

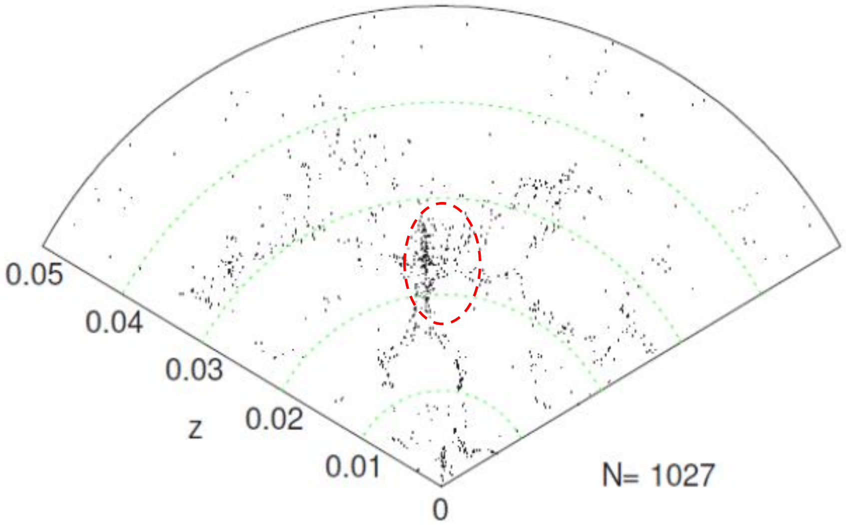
Define Position vector in redshift space as

$$\vec{s} \equiv \vec{X} + \mu\frac{v_{pec}}{H_0}\vec{n}, \quad \vec{n}\cdot\vec{v}_{pec} = \mu v_{pec}$$

$\vec{n}$:unit vector along the line of sight

Finger of God effect





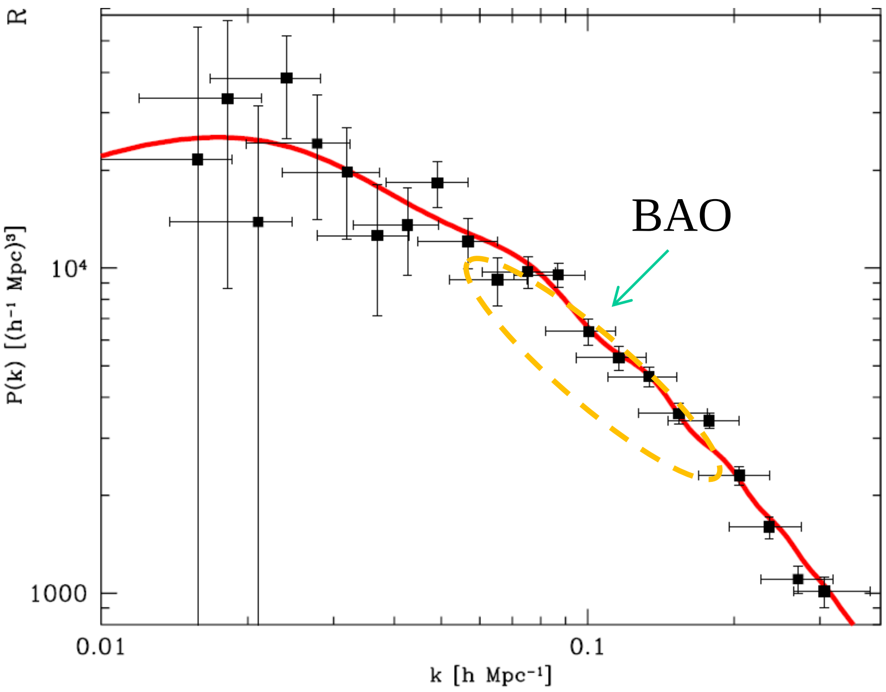
A.J.S. Hamilton, in "The Evolving Universe"

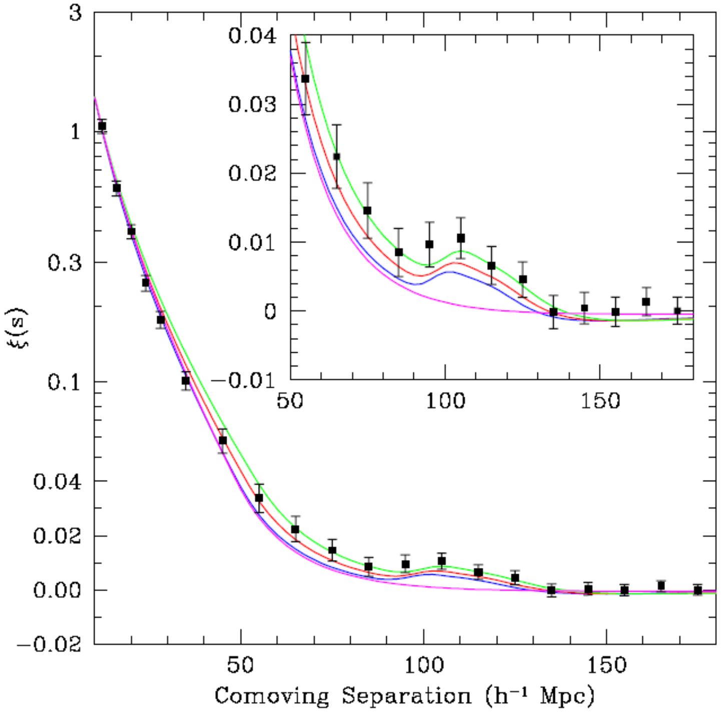
Baryon Acoustic Oscillation(BAO)

Relic of sound waves with wave length of the order of 150 Mpc in photon-baryon fluid before last scattering

Dense regions have a tendency to produce more galaxies

BAO using SDSS by Eisenstein et al. (2005)





$(\Omega_{m0}h^2, \Omega_{b0}h^2)$

$(0.12, 0.024)$

$(0.13, 0.024)$

$(0.14, 0.024)$

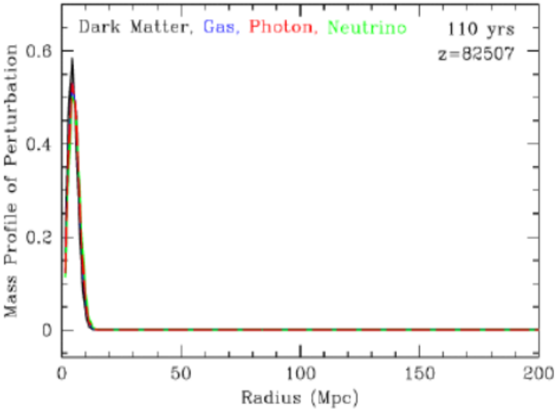
$(0.105, 0.0)$

green

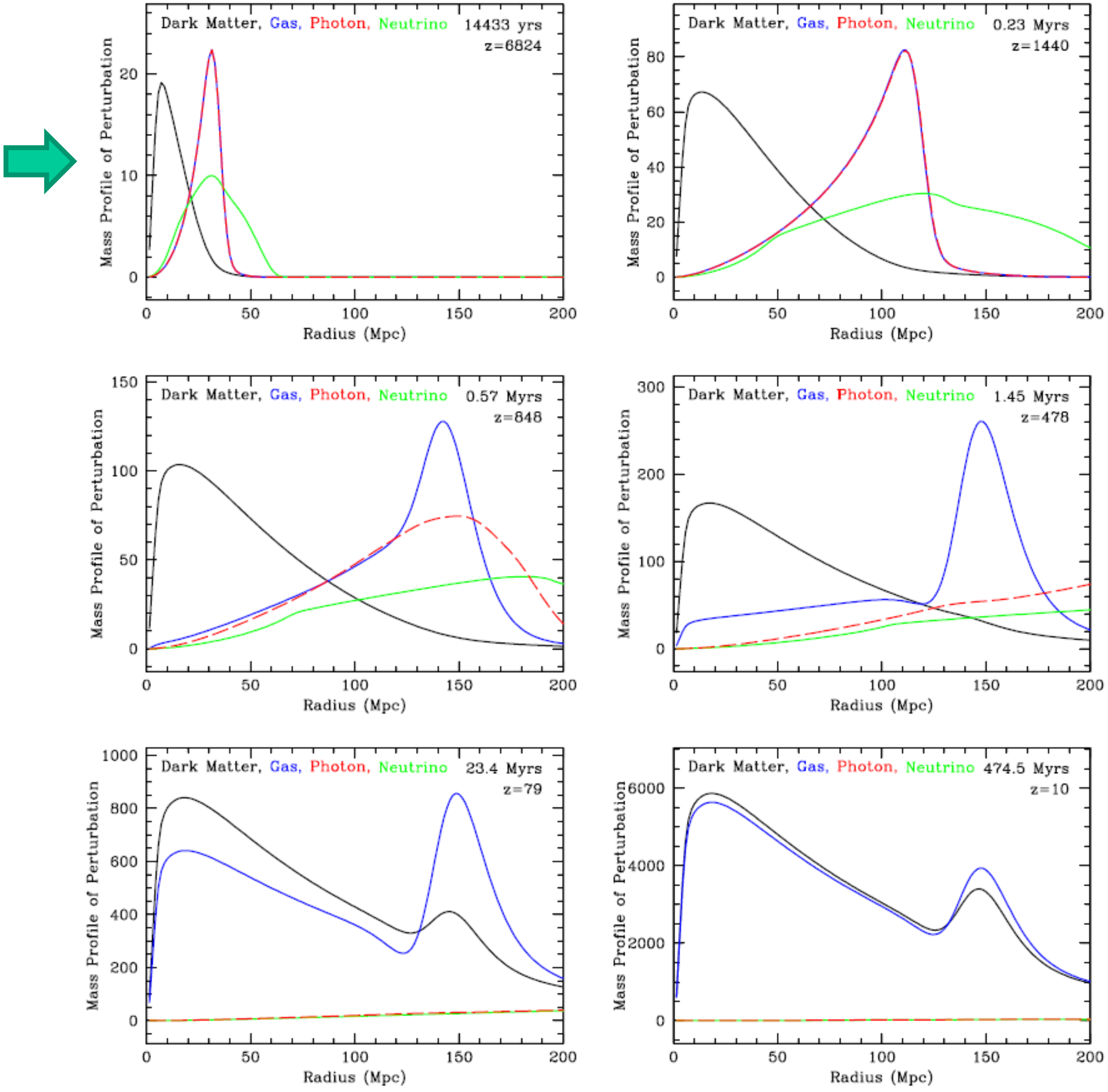
red

purple

Basic Mechanism of BAO



Einsenstein, Seo &White(2006)



BAO scale

Sound horizon

$$r_s(t) \equiv \int_0^t \frac{c_s}{a}\mathrm{d}t = \int_0^{a(t)} \frac{c_s}{a}\frac{\mathrm{d}t}{\mathrm{d}a}\mathrm{d}a = \int_0^{a(t)} \frac{c_s}{a^2 H}\mathrm{d}a$$

Sound velocity

$$c_s = \frac{c}{\sqrt{3}}\frac{1}{\sqrt{1+R}}$$

$$R \equiv \frac{\bar{\rho}_{\mathrm{b}} + \bar{p}_{\mathrm{b}}}{\bar{\rho}_\gamma + \bar{p}_\gamma} = \frac{3}{4}\frac{\bar{\rho}_{\mathrm{b}}}{\bar{\rho}_\gamma} = \frac{3}{4}\frac{\rho_{\mathrm{b}}(a_{\mathrm{eq}})}{\rho_\gamma(a_{\mathrm{eq}})}\frac{a}{a_{\mathrm{eq}}} = R_{\mathrm{eq}}\frac{a}{a_{\mathrm{eq}}}$$

Explicit calculation of the sound horizon

$$H^2 = \frac{8\pi G}{3c^2}(\bar{\rho}_{\mathrm{m}} + \bar{\rho}_\gamma)$$

$$a_{\mathrm{eq}} = a(t_{\mathrm{eq}}), R_{\mathrm{eq}} = R(t_{\mathrm{eq}})$$

$$\frac{8\pi G}{3c^2} = H_{\mathrm{eq}}^2 2\bar{\rho}_{\mathrm{m}}(a_{\mathrm{eq}}).$$

$$a^4 H^2 = H_{\mathrm{eq}}^2 a^4 \frac{\bar{\rho}_{\mathrm{m}}(a) + \bar{\rho}_\gamma(a)}{2\bar{\rho}_{\mathrm{m}}(a_{\mathrm{eq}})} = \frac{1}{2}H_{\mathrm{eq}}^2 a_{\mathrm{eq}}^4\left(1 + \frac{a}{a_{\mathrm{eq}}}\right).$$

$$k_{\mathrm{eq}} \equiv \frac{a_{\mathrm{eq}} H_{\mathrm{eq}}}{c}$$ ⇨ $$a^2 H = \frac{c a_{\mathrm{eq}} k_{\mathrm{eq}}}{\sqrt{2}}\sqrt{1 + \frac{a}{a_{\mathrm{eq}}}}$$

$$r_s(t) = \sqrt{\frac{2}{3}} \frac{1}{k_{\rm eq}\sqrt{R_{\rm eq}}} \int_0^R \frac{{\rm d}R'}{\sqrt{(R'+1)(R'+R_{\rm eq})}} = \frac{2\sqrt{2/3}}{k_{\rm eq}\sqrt{R_{\rm eq}}} \ln\left(\frac{\sqrt{R+1}+\sqrt{R+R_{\rm eq}}}{\sqrt{R_{\rm eq}}+1}\right)$$

where we used $\frac{{\rm d}a}{{\rm d}R} = \frac{a_{\rm eq}}{R_{\rm eq}}$

At the time of decoupling

$$r_s(t_{\rm dec}) = \frac{2\sqrt{2/3}}{k_{\rm eq}\sqrt{R_{\rm eq}}} \ln\left(\frac{\sqrt{R_{\rm dec}+1}+\sqrt{R_{\rm dec}+R_{\rm eq}}}{\sqrt{R_{\rm eq}}+1}\right).$$

$$R_{\rm dec} \equiv R(t_{\rm dec})$$

Numerical values

$$k_{\rm eq} = \frac{\sqrt{2}H_0}{c}\frac{\Omega_{\rm m0}}{\sqrt{\Omega_{\rm r0}}} = 9.514\times10^{-3}\left(\frac{\Omega_{\rm m0}h^2}{0.13}\right){\rm Mpc}^{-1}$$

$$R_{\rm eq} = \frac{3}{4}a_{\rm eq}\frac{\Omega_{\rm b0}}{\Omega_{\gamma0}} = \frac{3}{8}g_{*0}\frac{\Omega_{\rm b0}}{\Omega_{\rm m0}} = 0.2231\left(\frac{\Omega_{\rm b0}h^2}{0.023}\right)\left(\frac{\Omega_{\rm m0}h^2}{0.013}\right)^{-1}$$

$$R_{\rm dec} = \frac{a_{\rm dec}}{a_{\rm eq}} = \frac{3}{8}g_{*0}a_{\rm dec}\frac{\Omega_{\rm b0}}{\Omega_{\rm r0}} = 0.6404\left(\frac{1+z_{\rm dec}}{1090}\right)^{-1}\left(\frac{\Omega_{\rm b0}h^2}{0.023}\right)$$

➡ $r_s(t_{\rm dec}) = 147.6{\rm Mpc} = 103h^{-1}{\rm Mpc}$

Effect of neutrino in structure formation

The present mass density of non-relativistic neutrino

$$\rho_\nu^{\rm nr} = \sum_{i=1}^{N_\nu^{\rm nr}} m_{\nu,i} n_{\nu,i},$$

The number density of each species

$$n_\nu \;=\; \frac{3\zeta(3)}{2\pi^2} T_\nu^3 \simeq\; 112(1+z)^3\ {\rm cm}^{-3},$$

$$T_\nu = (4/11)^{1/3} T_{\gamma 0}(1+z),$$

$$T_{\gamma 0} = 2.725\,K \;\rightarrow\; T_{\nu 0} \approx 1.9\,K$$

The density parameter for massive neutrino

$$\Omega_\nu \;\equiv\; \frac{8\pi G \rho_\nu^{\rm nr}}{3H_0^2} = \frac{8\pi G n_\nu}{3H_0^2} \sum_{i=1}^{N_\nu^{\rm nr}} m_{\nu,i} \simeq \frac{\sum_i m_{\nu,i}}{94.1 h^2\ {\rm eV}}.$$

Non-relativistic epoch

Neutrino becomes non-relativistic when its mean energy is equal to the rest mass energy

The mean energy per particle

$$\langle E \rangle = \frac{7\pi^4}{180\zeta(3)} T_\nu \simeq 3.15 T_\nu,$$

Thus

$$T_{\nu,i}^{\rm nr} \equiv \frac{180\zeta(3)}{7\pi^4} m_{\nu,i} \simeq 3680 \left(\frac{m_{\nu,i}}{1\ {\rm eV}}\right)\ {\rm K}.$$

$$1 + z_{{\rm nr},i} \simeq 1890 \left(\frac{m_{\nu,i}}{1\ {\rm eV}}\right)$$

The comoving wave number corresponding to the Hubble horizon size at z_{nr}

$$k_{{\rm nr},i} \equiv \frac{H(z_{{\rm nr},i})}{1 + z_{{\rm nr},i}} = \frac{\Omega_m^{1/2} h (1 + z_{{\rm nr},i})^{1/2}}{2998\ {\rm Mpc}}$$

$$\simeq 0.0145 \left(\frac{m_{\nu,i}}{1\ {\rm eV}}\right)^{1/2} \Omega_m^{1/2} h\ {\rm Mpc}^{-1}.$$

Neutrino free streaming scale

$$L_{fs}(z) \approx \frac{v_\nu}{H(z)} \Rightarrow k_{fs} \approx \frac{H(z)}{v_\nu}$$

Detailed calculation shows

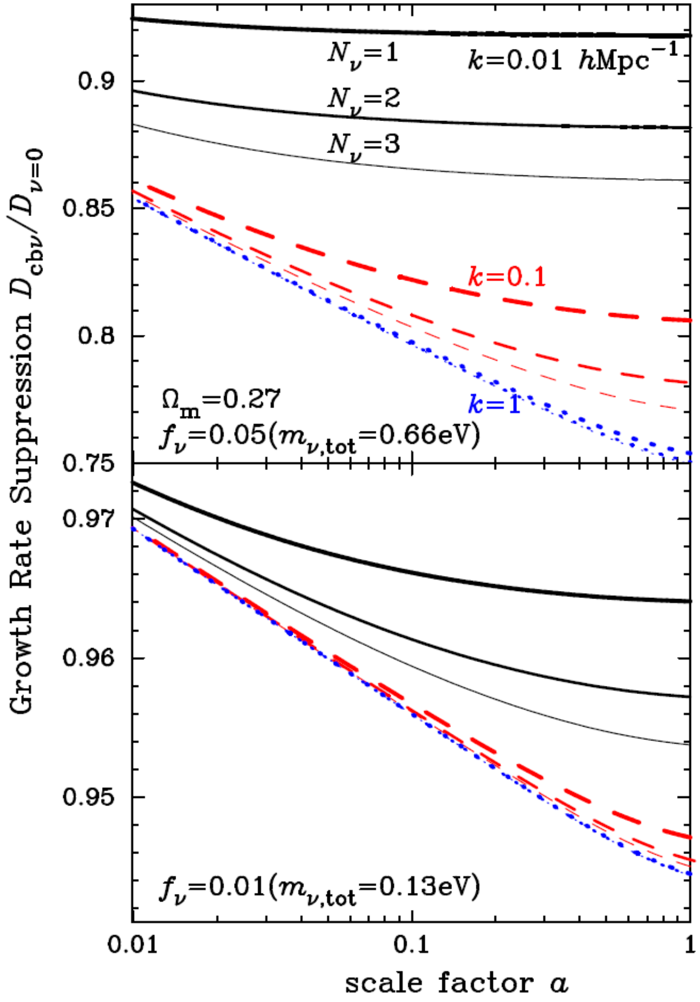
$$k_{\mathrm{fs},i}(z) \simeq \frac{0.677}{(1+z)^{1/2}} \left(\frac{m_{\nu,i}}{1\ \mathrm{eV}}\right) \Omega_{\mathrm{m}}^{1/2} h\ \mathrm{Mpc}^{-1}.$$

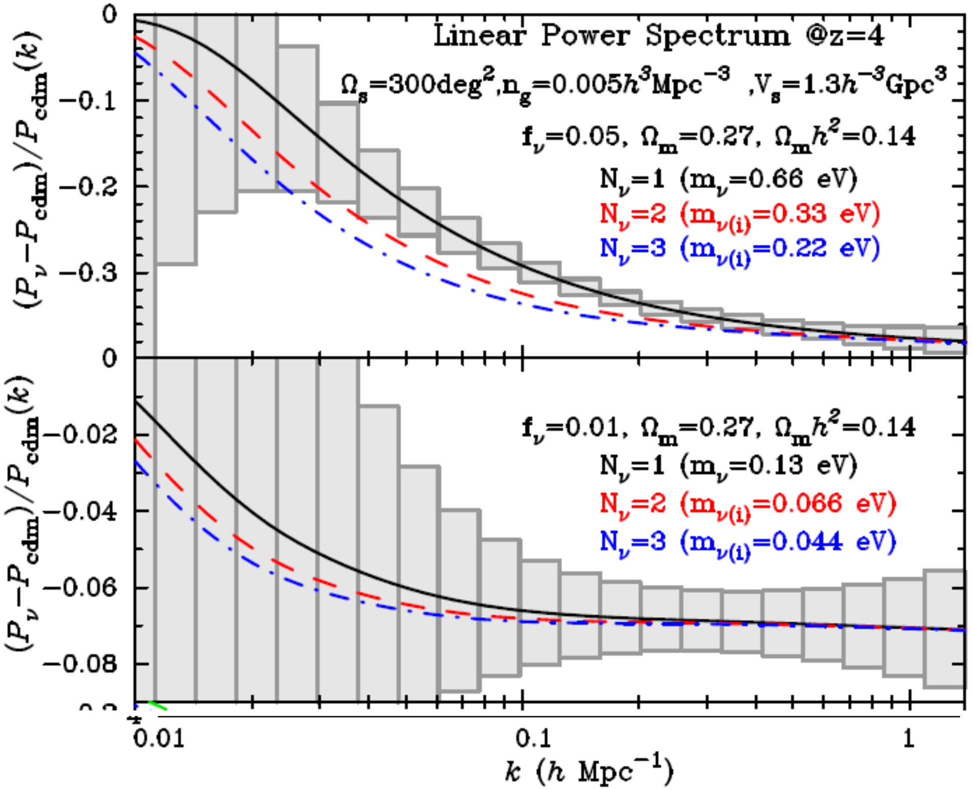
$k_{fs} = 0.09 - 0.179\ Mpc^{-1}$at z=1-6 for m_ν=1 eV

For scales larger than free streaming $k < k_{fs}$, neutrinos can cluster and fall into gravitational potential well together with CDM and baryon

For scales smaller than free streaming $k > k_{fs}$, neutrinos escape from potential well, and the growth of matter perturbation is slowed down relative to that on the larger scales.

The matter power spectrum for $k > k_{fs}$ is suppressed relative to that for $k < k_{fs}$





Takada, Komatsu.& Futamase(2006)

From galaxy survey we know the following

- Structure i

  Galaxy, group of galaxies, cluster of galaxies, supercluster
- Bottom-up rather than top-down

  Small objects form earlier and they attract each other o for larger objects
- CDM rather than HDM
- Even if HDM is not dominant component of dark matter, it has an important effect in structure formation,
- Galaxies distribute homogeneously over the scale of 100 Mpc.
- WDM(Warm dark matter) is not totally eliminated as a candidae of dark mater

1.4　CMBの発見とその意味

~ 1946　ビッグバン（超高温、超高密度状態）の名残としてプランク分布を持った放射が宇宙をくまなく満たしていることが予言される

1965　波長7.35 cmで宇宙をくまなく満たす絶対温度約3度の放射が発見される（A. Penzian& R. Wilson)

波長3.2 cmの観測からこの放射が絶対温度約3度の黒体放射であることが確認される　(P.Roll&D.Wilkinson)

1989　COBE(Cosmic Background Explorer)がCMBが絶対温度約2.725度の完全な黒体放射であることを確認

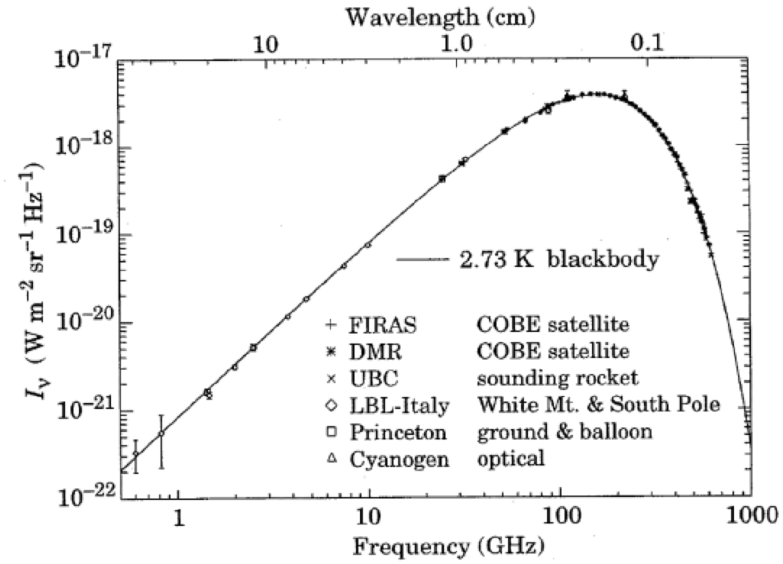
$$I_\nu = \frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/kT_{CMB})-1}$$

$$\rho_{CMB} = 4.17\times10^{-13}\left(\frac{T_{CMB}}{2.725\ \mathrm{K}}\right)^4 \mathrm{erg\ cm^{-3}}$$

$$n_{CMB} \approx 415/cm^3$$

COBE also shows the isotropy

$$\left|\frac{\Delta T}{T}\right| \le 10^{-5}$$



CMB のスペクトル．実線は，観測データを最も良く再現する黒体放射スペクトル (Particle Data Group, 2004, “Review of Particle Physics”, Physics Letters B, 592, 1)

この発見の意味

放射エネルギー密度 $\rho_r \propto a^{-4}$

物質エネルギー密度 $\rho_M \propto a^{-3}$

➡ 放射・物質等時期 $\rho_r(t_{eq}) = \rho_m(t_{eq})$

$$\frac{\rho_r(t_0)}{a^4(t_{eq})} = \frac{\rho_m(t_0)}{a^3(t_{eq})}$$

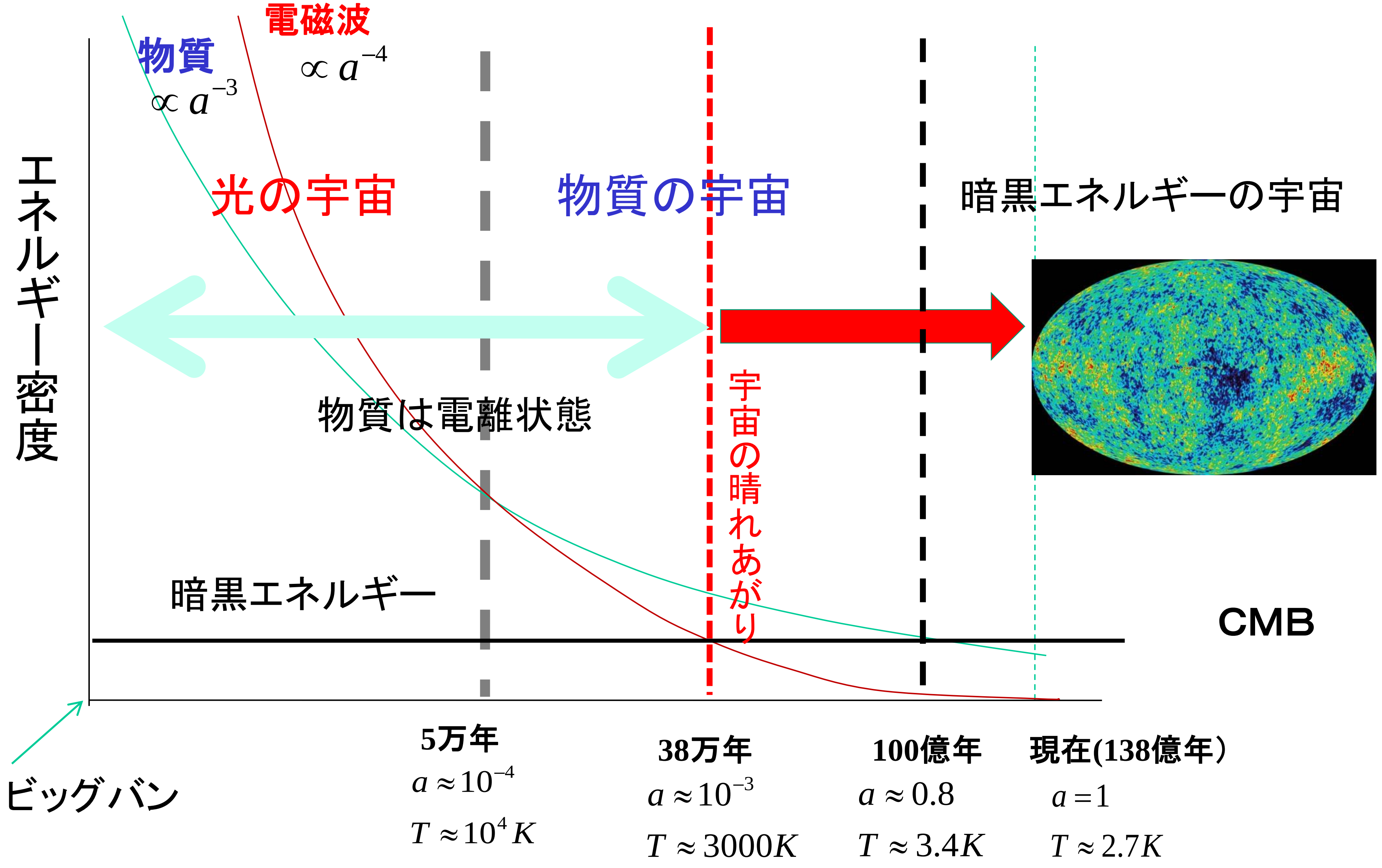
➡ $$a(t_{eq}) = \frac{\rho_r(t_0)}{\rho_m(t_0)} = \frac{\rho_r(t_0)/\rho_{cr}}{\Omega_{m0}} = \frac{4.18\times10^{-5}}{\Omega_{m0}h^2} \approx 3\times10^{-4}$$

$$T_{eq} = T_0 / a_{eq} \approx 10^4 K$$

$a < a_{eq}$ Radiation Dominant（RD) (放射優勢)

$a > a_{eq}$ Matter Dominant（MD) (物質優勢)

宇宙膨張によるエネルギー密度の変化



物質
$\propto a^{-3}$

電磁波
$\propto a^{-4}$

エネルギー密度

光の宇宙

物質の宇宙

暗黒エネルギーの宇宙

物質は電離状態

宇宙の晴れあがり

暗黒エネルギー

CMB

ビッグバン

5万年
$a \approx 10^{-4}$
$T \approx 10^{4} K$

38万年
$a \approx 10^{-3}$
$T \approx 3000K$

100億年
$a \approx 0.8$
$T \approx 3.4K$

現在(138億年）
$a = 1$
$T \approx 2.7K$

光子-バリオン流体

トムソン散乱による平均自由行程

$$\lambda \approx \frac{1}{n_e \sigma_T} \approx 2.25\times 10^{6} (1+z)^{-3}\ \mathrm{kpc}$$

ここで

$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right) \approx 6.68\times 10^{-25}\ \mathrm{cm}^2$$

トムソン散乱断面積

$$n_e = n_p = \frac{\rho_b}{m_p} = \frac{\rho_{b0} a^{-3} / \rho_{cr0}}{m_p / \rho_{cr0}} \approx 1.12\times 10^{-5}\,\Omega_{b0} h^2 (1+z)^3$$

地平線スケール（宇宙の大きさの目安）

$$R_H(z) = \frac{c}{H(z)} \approx \frac{2c}{H_0 \Omega_{m,0}^{1/2}} (1+z)^{-3/2} \approx \frac{6000}{h\Omega_{m,0}^{1/2}} (1+z)^{-3/2}\ \mathrm{Mpc}$$

⇨

$$\frac{\lambda(z)}{R_H(z)} \approx 500 h \Omega_{m,0}^{1/2} (1+z)^{-3/2} \approx 0.0007\ \mathrm{at}\ \mathrm{z} = 3400$$

光子・バリオン流体における音速

- 音速

$$c_s^2 = \frac{dP}{d\rho} = \frac{dP_\gamma}{d(\rho_b + \rho_\gamma)} = \frac{d(1/3\rho_\gamma)}{d(\rho_b + \rho_\gamma)} = \frac{1}{3}\frac{1}{1+R}$$

$$R = \frac{3\rho_b}{4\rho_\gamma} = R_{eq}\frac{a}{a_{eq}},$$

$$R_{eq} = \frac{3\Omega_{b0}}{4\Omega_{\gamma 0}} a_{eq} \approx 0.1971\left(\frac{\Omega_{b0}h^2}{0.022}\right)\left(\frac{\Omega_{m0}h^2}{0.14}\right)^{-1}$$

- 音波地平線の広がり

$$r_s(t) = \int_0^t \frac{c_s}{a} dt$$

$$r_s(t_{dec}) \approx 103\, h^{-1}\mathrm{Mpc} \approx 147.6\,\mathrm{Mpc}$$

宇宙の晴れあがり

- 陽子の電子捕獲がいつ起こるか

$$p+e^{-}\rightarrow H+\gamma$$

エネルギー的には宇宙の温度が水素の結合エネルギー

$$B=m_p+m_e-m_H=13.6\,\mathrm{eV}\approx 15000\,\mathrm{K}$$

程度に下がれば、中性化が起こるが、

宇宙のバリオン・光子数比

$$\eta=\frac{n_b}{n_{CMB}}\approx 6\times 10^{-10}$$

バリオン１個に対して数十億個の光子がある

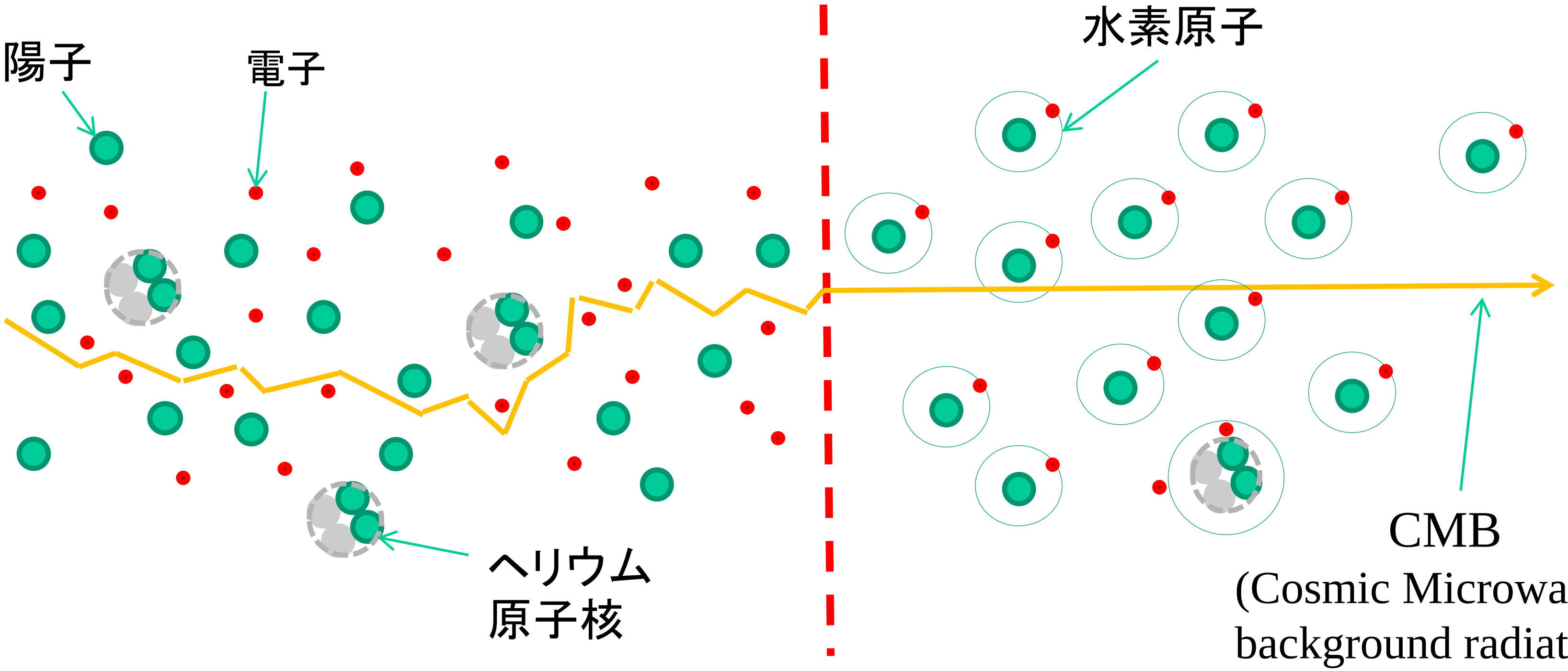
宇宙の温度が水素の結合エネルギー程度に下がっても、まだ莫大な数の高エネルギー光子が存在し、できた水素を電離してしまう。

宇宙の温度が3000度程度に下がると、13.6eV以上のエネルギーをもった光子がバリオン1個あたり1個以下になるので、中性化が進行する。

CMBの起源

光は荷電粒子と頻繁に衝突して直進できない

温度が下がり陽子は電子を捕獲して中性の水素原子、ヘリウム原子となり、光は物質と衝突することなく直進する



超高温超高密度状態
素粒子の世界

陽子

電子

ヘリウム
原子核

水素原子

CMB
(Cosmic Microwave background radiation)

10億K
3分

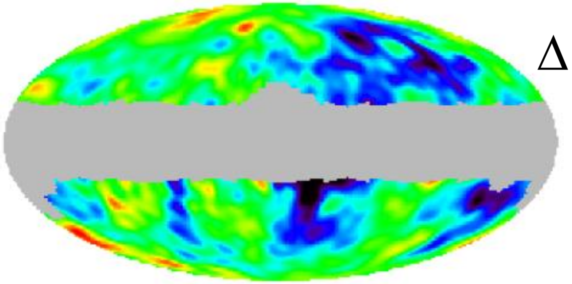
3000K
38万年

CMB 温度揺らぎの発見

1992年、COBEがCMBの温度揺らぎを発見

$$\frac{\delta T}{T_0}(\Delta\theta) = \frac{T(\vec{n}_1) - T(\vec{n}_2)}{T_0} \simeq 10^{-5}$$

COBE



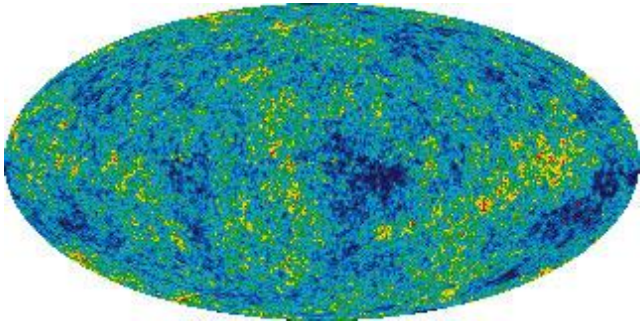
$\Delta\theta \geq 7^\circ$

COBEに比べ、

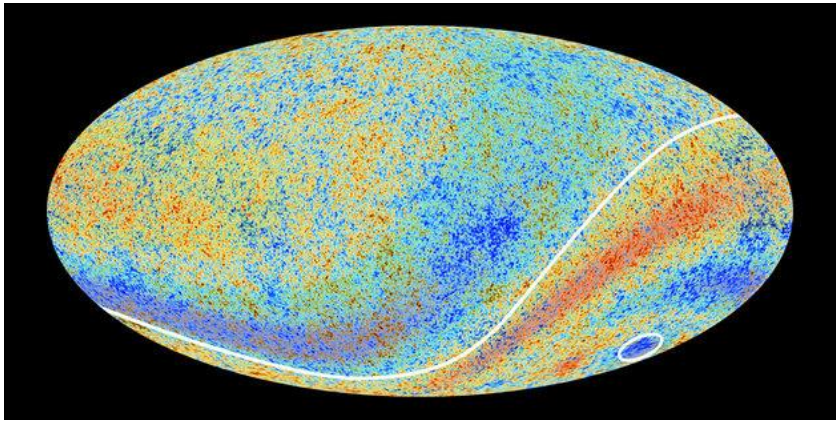
●角度分解能で３５倍

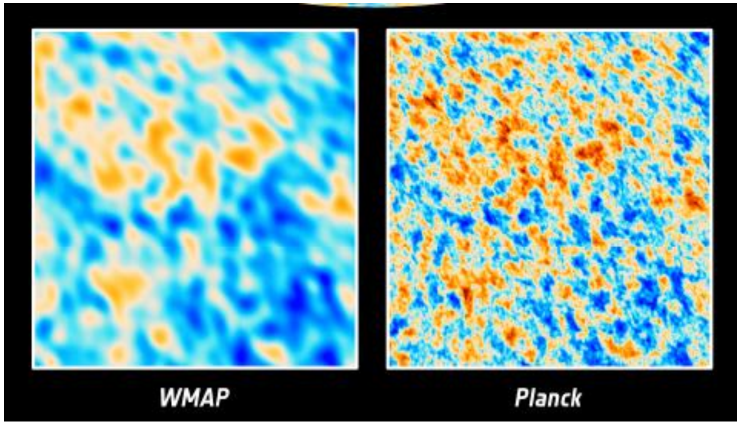
●感度で１０倍の改善

2004　WMAP　$\Delta\theta \geq 12'$



2013　Planck　$\Delta\theta \geq 5'$





CMBの温度揺らぎの原因

- 光子・バリオン流体に起こる音波（粗密波）

揺らぎの典型的な角度スケールは、音波地平線のスケールで決まる

音波地平線の長さは、宇宙の晴れ上がりまでに音波が走った距離

音波の速度×晴れ上がり**の宇宙時間**

$$c_s^2 = \frac{1}{3}\frac{1}{1+R} \qquad R = \frac{3\rho_b}{4\rho_\gamma}$$

⇨ $$r(t_{dec}) \approx 147.6\,\mathrm{Mpc}$$

音波の速度は、流体中の光子とバリオンの割合で決まる

現在から見る角度スケール

$$\theta_{SH} \cong \frac{\lambda_{LS}}{D} \approx 1^o$$

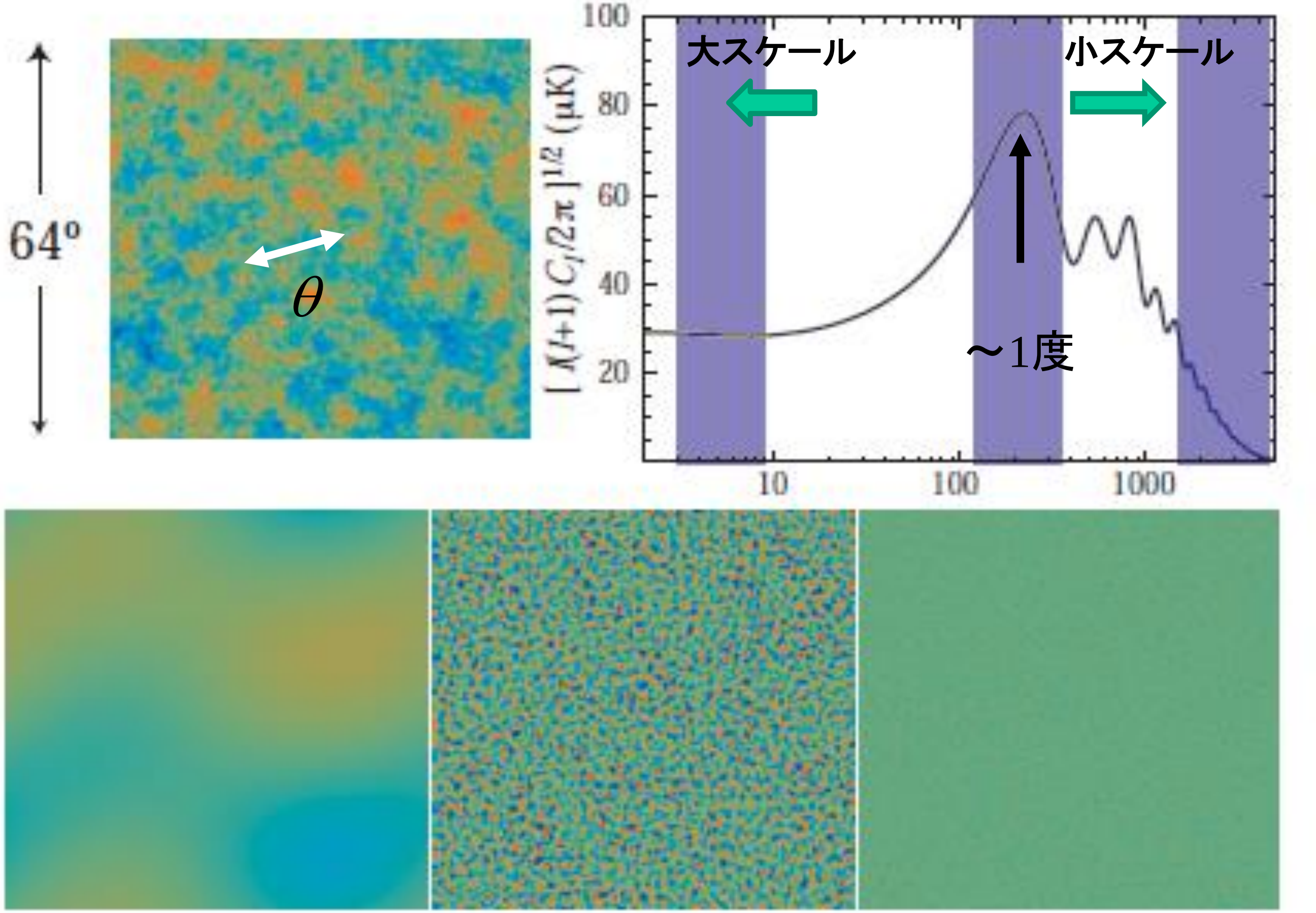
D は最終散乱面までの距離

$$D(t) = \int_{t_{dec}}^{t_0} \frac{dt}{a}$$

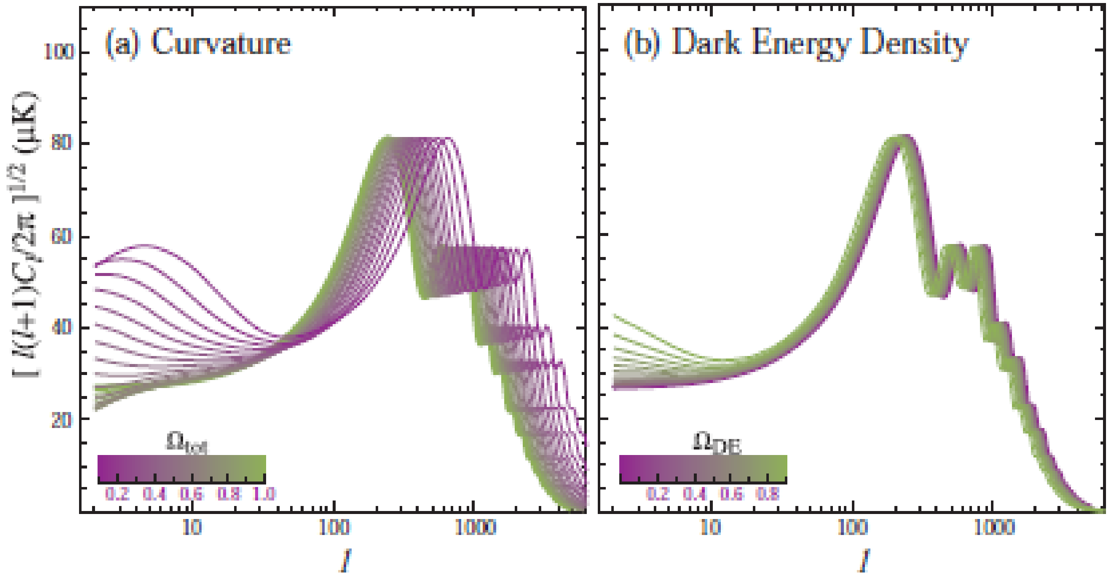
温度揺らぎのパワースペクトル

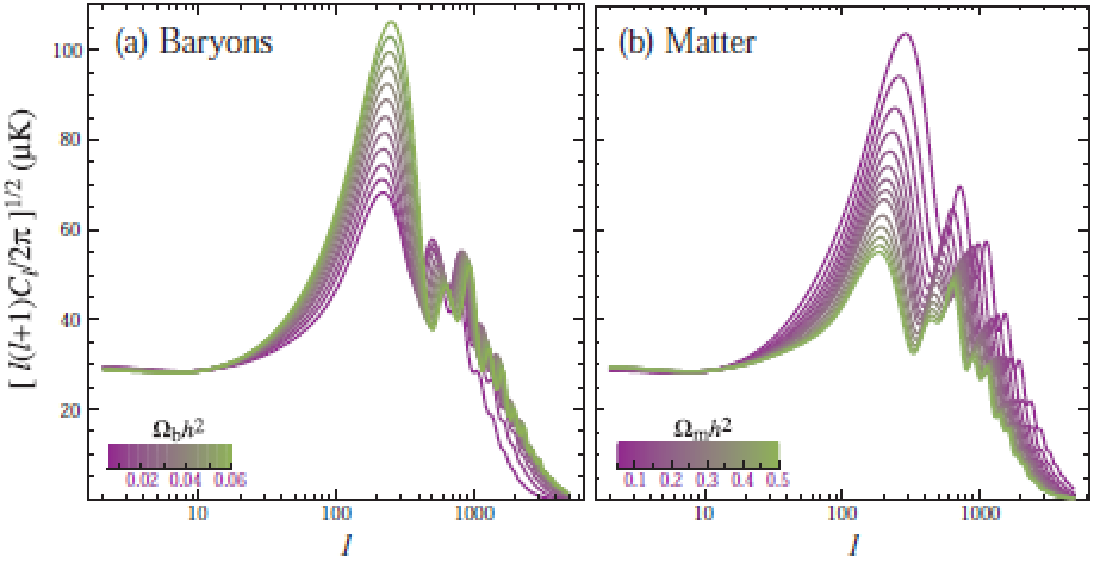
2点相関関数

$$\frac{\delta T(\theta,\phi)}{T} = \sum_{\ell,m} a_{\ell m} Y_{\ell m}(\theta,\phi), \qquad C_\ell = \left\langle a_{\ell m} a^{*}_{\ell m} \right\rangle \qquad \ell \approx \frac{180^\circ}{\theta}$$

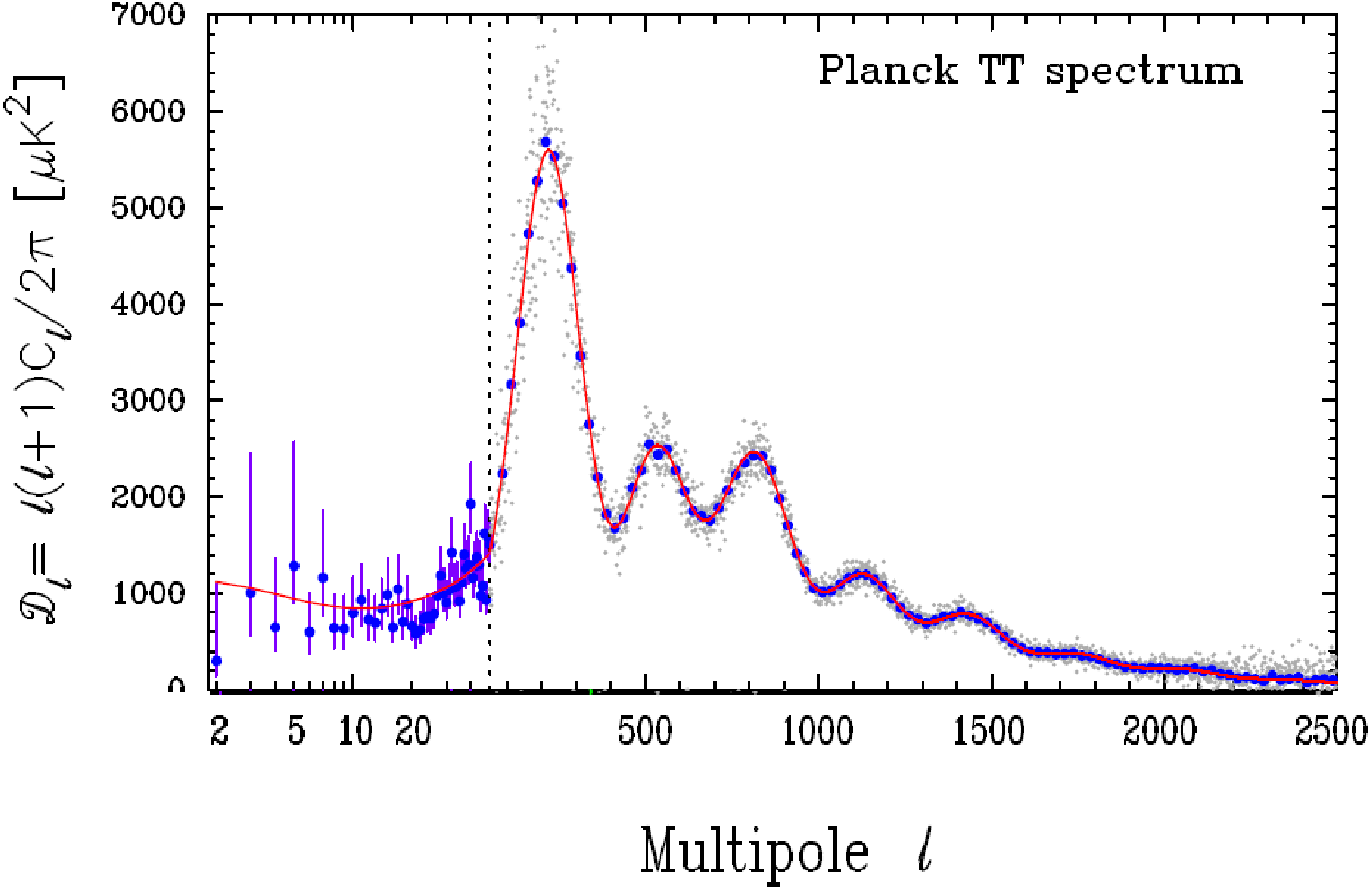


Cosmological parameter dependence on the power spectrum C_l





Power spectrum obtained by Planck Satelite



- 宇宙を記述するパラメータは以下の通り

$$H_0 = 67.9 \pm 1.5\,\mathrm{km/s/Mpc},\ (h = 0.679 \pm 0.015)$$

$$t_0 = (13.796 \pm 0.058) \times 10^{10}\ \mathrm{yrs}$$

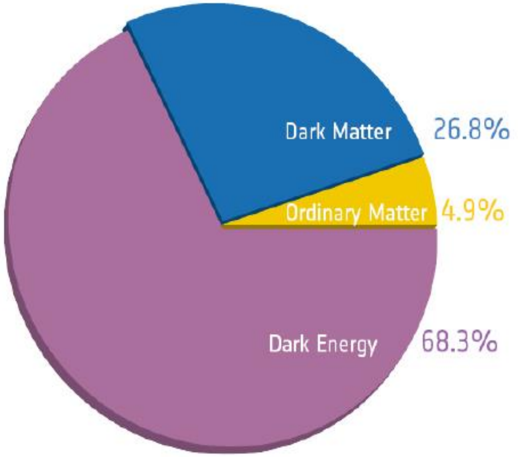
$$\Omega_{\Lambda,0} = 0.693 \pm 0.019,$$

$$\Omega_{CDM,0} h^2 = 0.1186 \pm 0.0031,$$

$$\Omega_{b0} h^2 = 0.02217 \pm 0.00033,$$

$$\Omega_{tot} \cong 1$$

Planck Results(arXiv:1405.0439, The Planck mission, F. R. Bouche

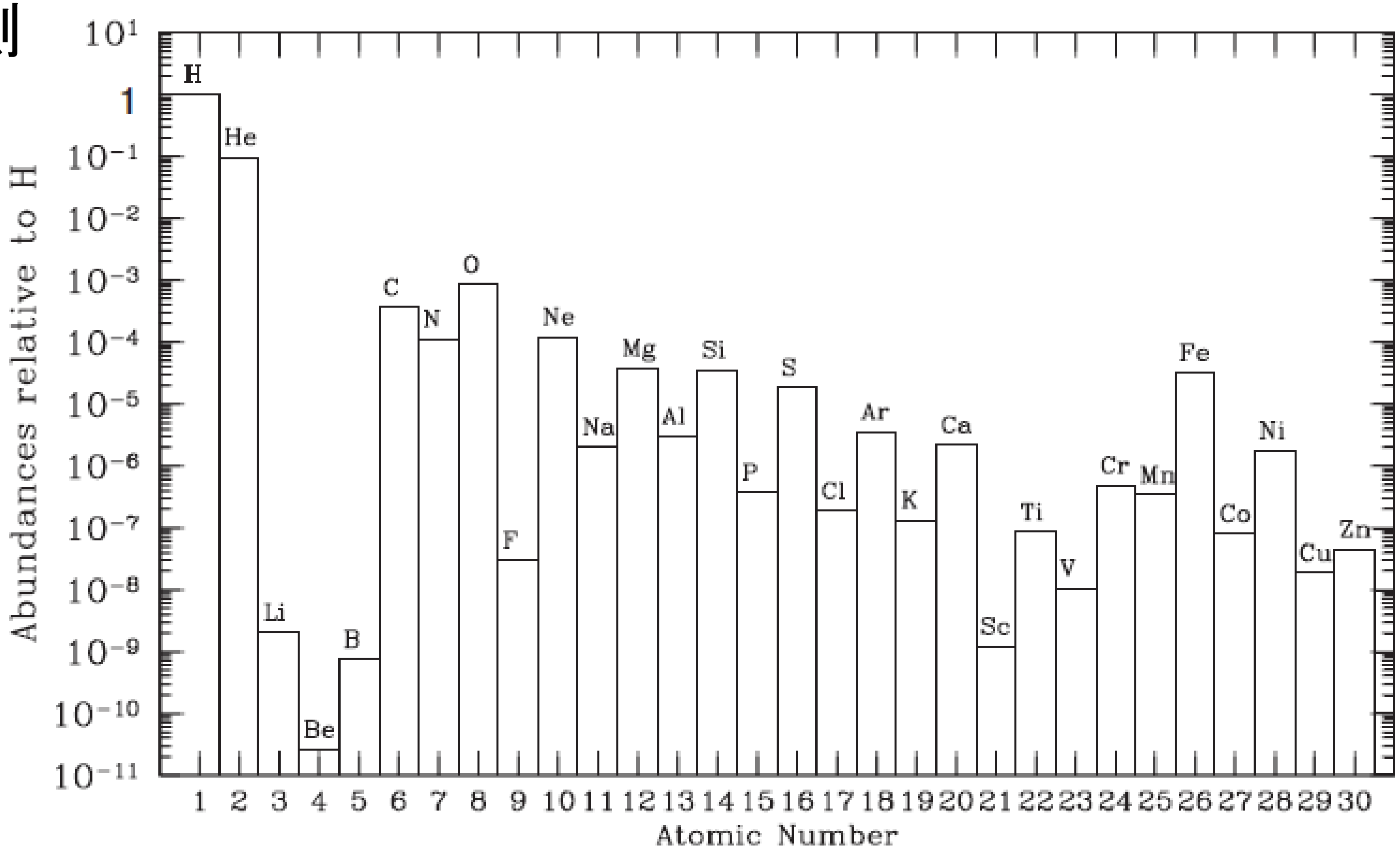


プランク

1.5　初期宇宙における軽元素合成とその観測

宇宙に存在する元素はどこでできたのか？

観測



太陽近傍の元素組成．縦軸は，水素の個数密度を 1 とした個数密度の割合を示す (出典：理科年表のデータに基づいて作図).

星の中でヘリウムは作られるが、小質量星（～0.5太陽質量以下）の場合作られたヘリウムはそのまま縮退して白色矮星となるが、寿命が長いため現在でもまだ水素の核融合が続いている状態で、星の外にはヘリウムはでていない

太陽程度より重たい星は、水素燃焼後にできるヘリウムコアは核融合反応を起こして炭素、窒素、酸素などに変わるので、ヘリウムはほとんど残らない

観測されるヘリウム量は粒子数比で10%（重量比で約25%）は多すぎる

➡ **星ができる前からヘリウムが存在していた**

宇宙の初めからヘリウムが存在していた！

ビッグバン理論では、宇宙初期に超高温、超高密度状態が実現される

1946　Gamow

中性子からできた始原物質が存在

$$n \to p + e^- + \bar{\nu}_e$$

できた陽子と中性子が融合してヘリウムを作る

$$2n + 2p \to {}^4He$$

1950　Hayashi

宇宙初期は弱い相互作用によって陽子と中性子は熱平衡にあった（ほぼ同数存在）

$$\nu_e + n \leftrightarrow p + e^-$$

$$e^+ + n \leftrightarrow p + \bar{\nu}_e$$

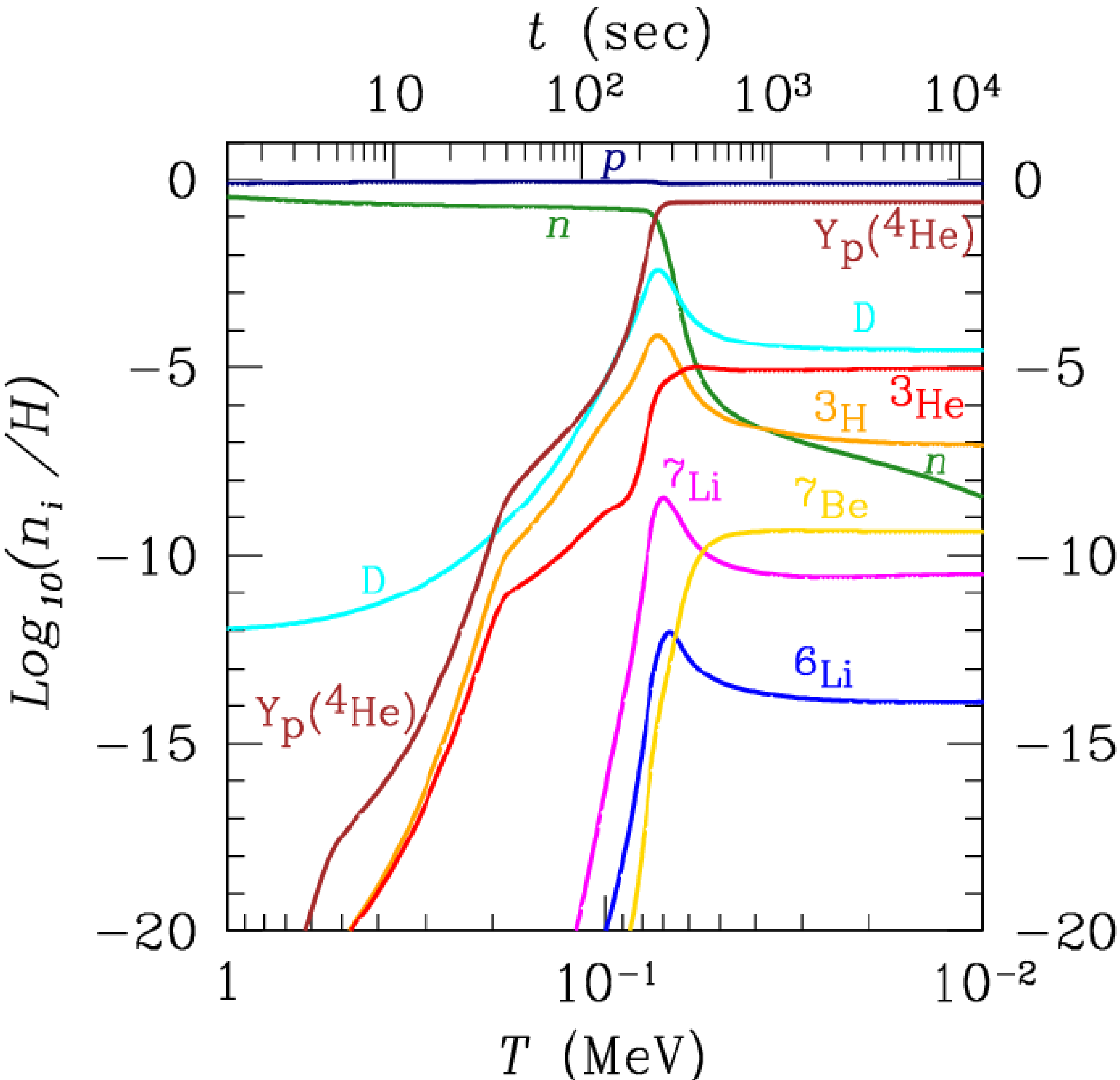
$$n \leftrightarrow p + e^- + \bar{\nu}_e$$

⇒

$$\left(\frac{n}{p}\right)_{\text{平衡}} = \exp\left(-\frac{Q}{kT}\right)$$

$$Q = m_n - m_p = 1.293\,\mathrm{MeV}$$

生成される軽元素の時間変化



$\eta_{10} \equiv 6.2$

の場合の
計算

観測

- ヘリウム4
  銀河系外の重元素量の少ない電離水素ガス領域（HII領域）
  大質量星からの紫外線によってHII領域内の水素とヘリウムが電離し、それが再結する際に放射される光を観測することで、水素とヘリウムの数密度の比が求められる

$$Y \approx 0.2449 \pm 0.0040$$ ）

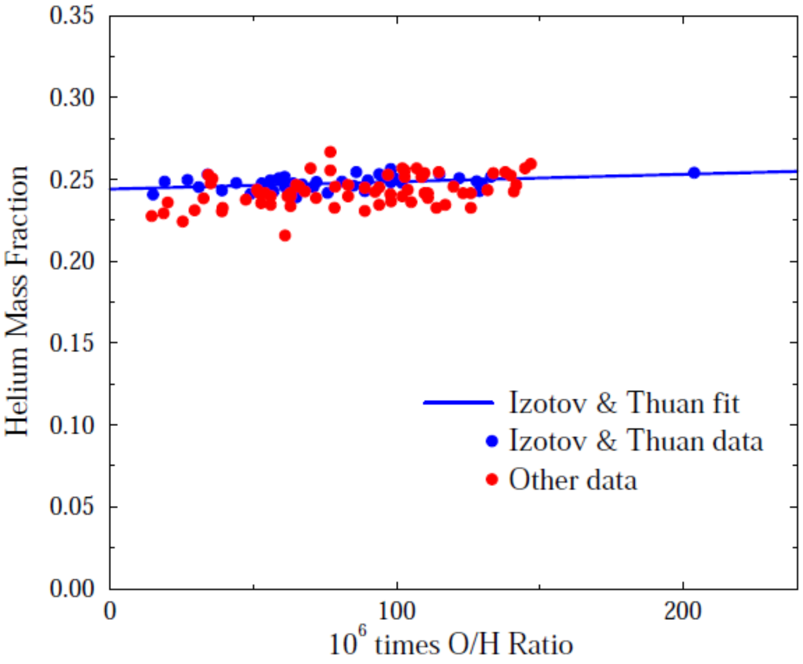
- 重水素
  遠方のクェーサーの吸収線

$$D/H = (2.53 \pm 0.04) \times 10^{-5}$$

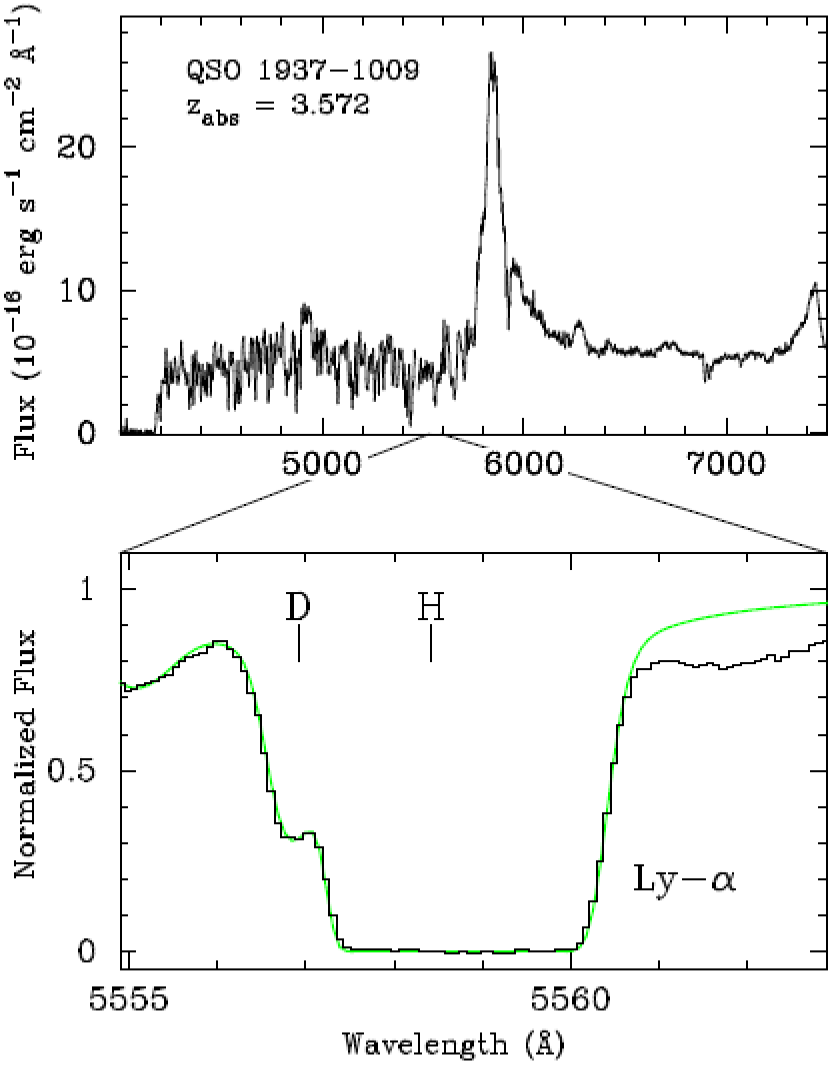
- リチウム
  銀河系の球状星団中の種族IIの重元素の少ない小さな星のスペクトルの観測

$$^{7}Li/H = (1.1 - 1.5) \times 10^{-10}$$

Observations

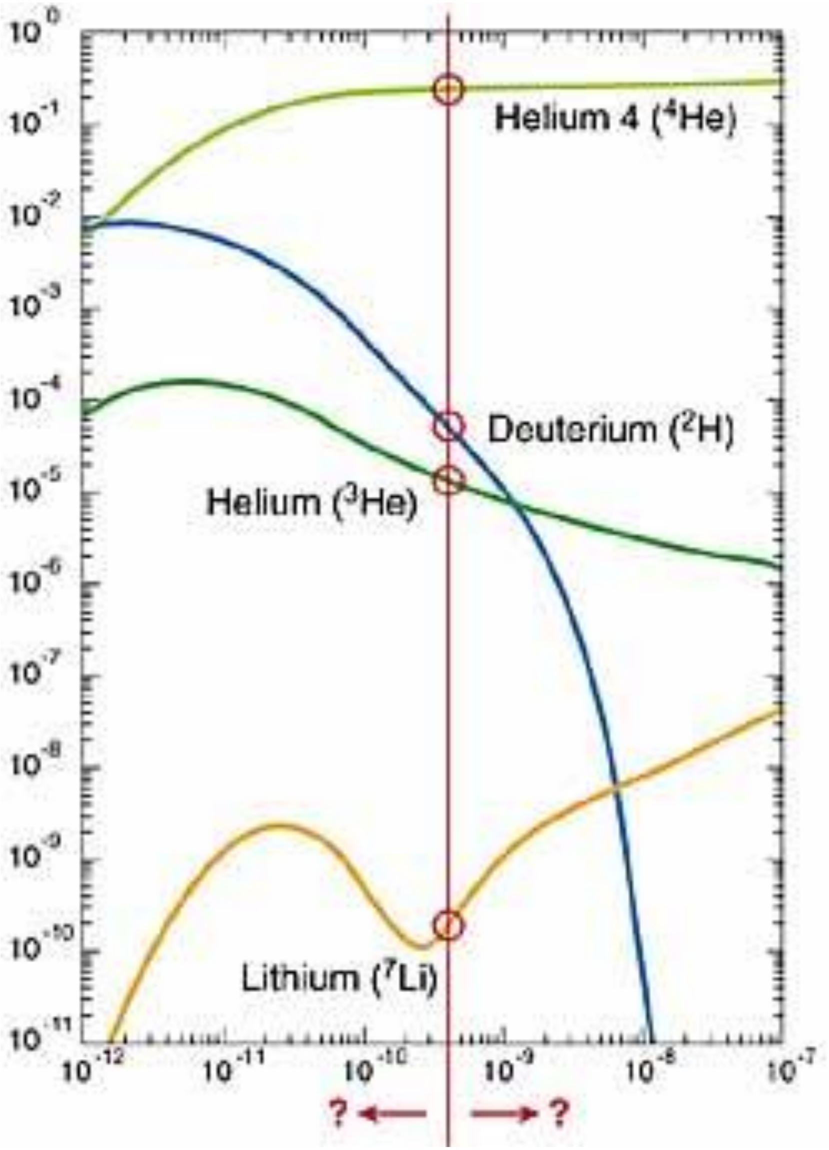


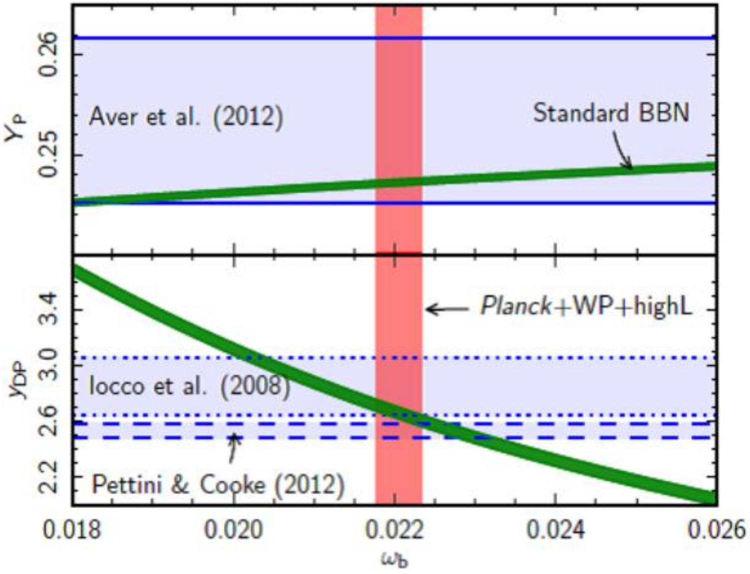
Measurements of He/H ratio in regions of hot, ionized gas (HII regions) in other galaxies.
Oxygen abundance is an indicator of steller processing



D observation in QSO spectrum

理論予想と観測

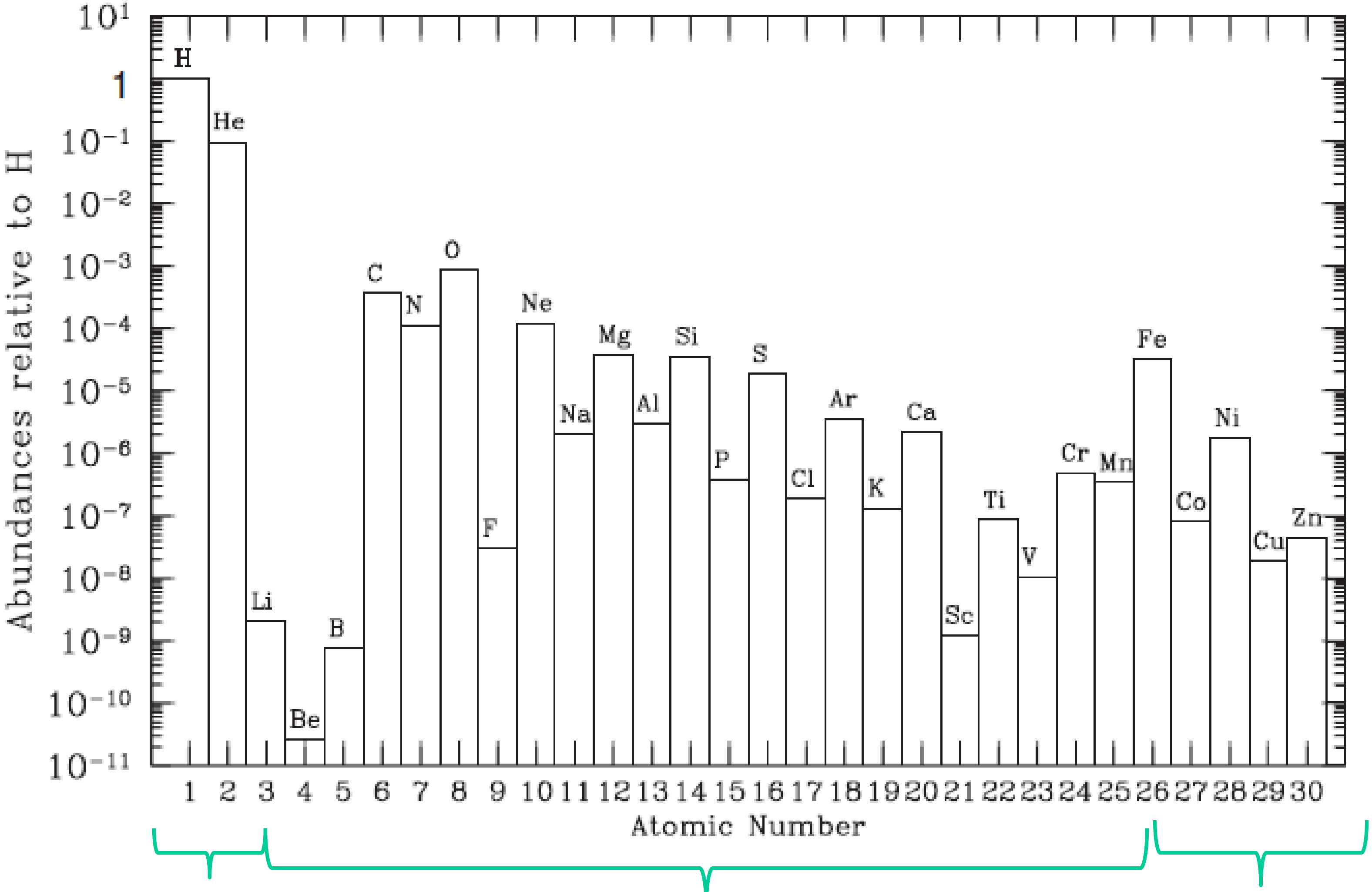




ヘリウム（上），重水素（下）の観測と理論の比較．横軸はバリオン密度 $\omega_b = \Omega_b h^2$．緑の範囲が理論予想で幅は核反応率の不定性による．薄い青の範囲が観測値で幅は観測誤差．縦の赤の範囲がプランク衛星などで得られたバリオン密度の範囲, PLANCK Collaboration, 2013

$$\frac{n_b}{n_\gamma} = (5.96 \pm 0.28) \times 10^{-10}$$

$$\Omega_{b0} = 0.0218 \pm 0.001$$



初期宇宙

星の中心核

超新星
ガンマ線バースト
（中性子連星系の
合体）

現代の観測のまとめ

- 宇宙は膨張しており、100Mpc程度で平均化すると空間は一様で等方
- 空間は平坦
- 宇宙は超高温、超高密度状態から始まった（ビッグバン）
- 宇宙はビッグバンから約5万年(z〜3400)まで放射優勢だった
- ビッグバンから38万年、温度が3000度のとき宇宙は晴れ上がり物質は中性化した
- 宇宙はビッグバンから約100億年後 (z~0.29) から加速膨張を始めた
- 宇宙における構造は小さなものが最初にでき、それらが集合・合体してより大きなものが出来上がった
- 暗黒物質は冷たい暗黒物質である