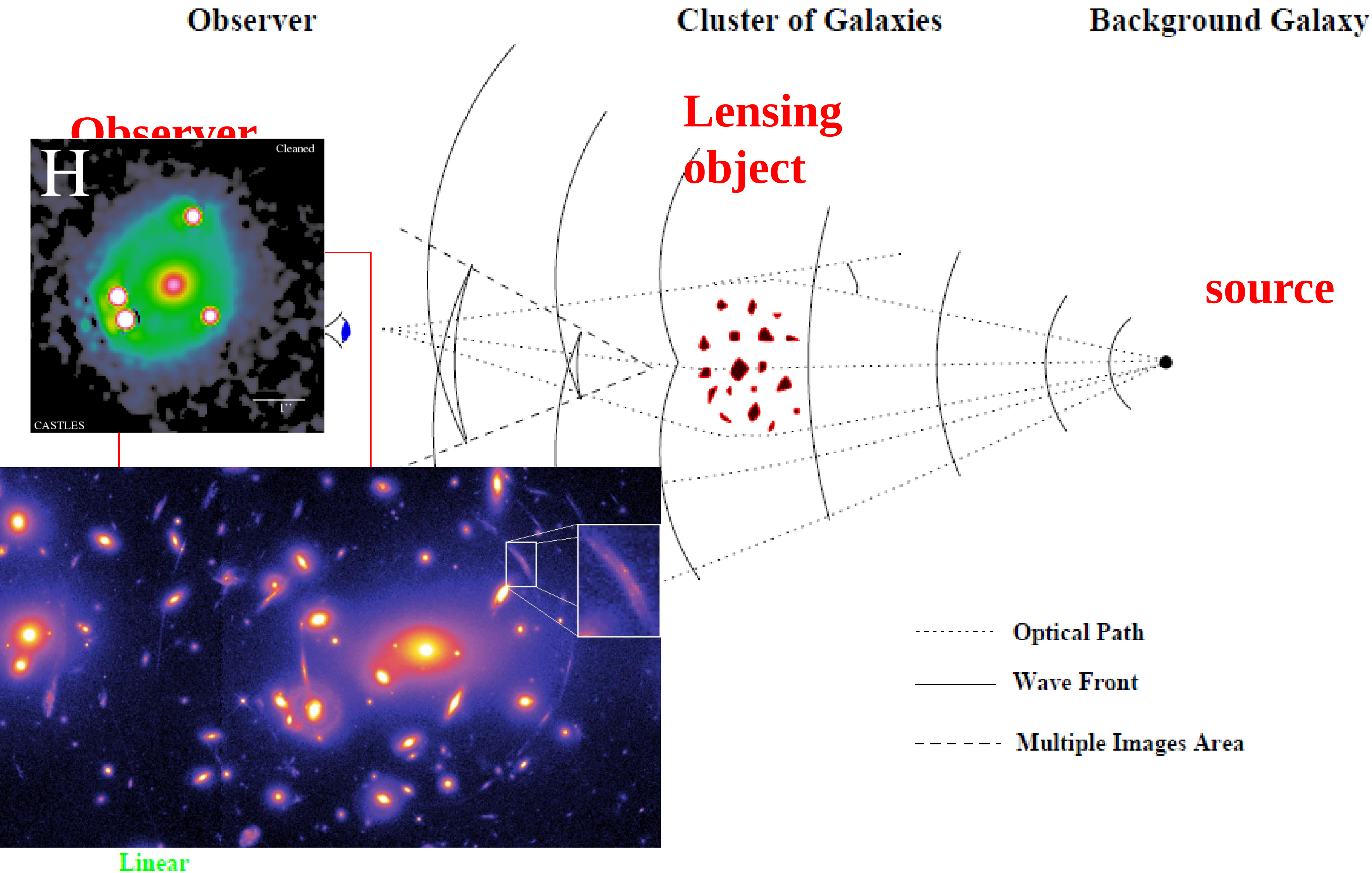
Gravitational Lens

Context

- GL equation and its properties
- Simple Lens model
- Strong Lens observables
- Weak Lensing

What is gravitational lensing?



Why gravitational lensing is useful?

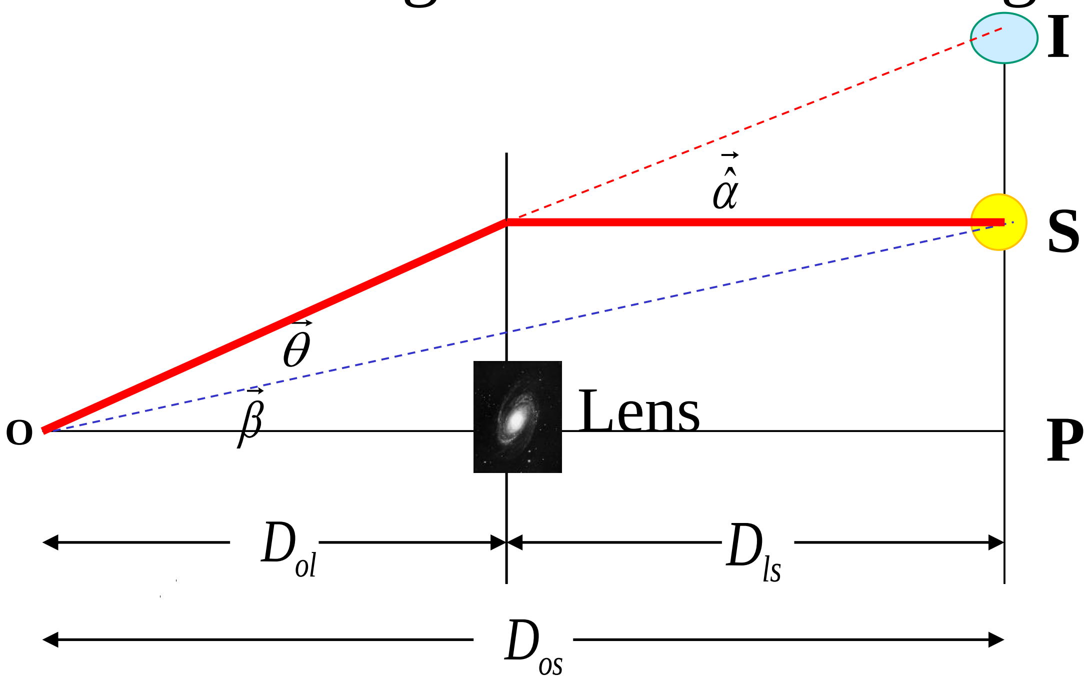
(1) Gravitational lensing is a unique method to study masses of cosmic structures independent of <M/L>

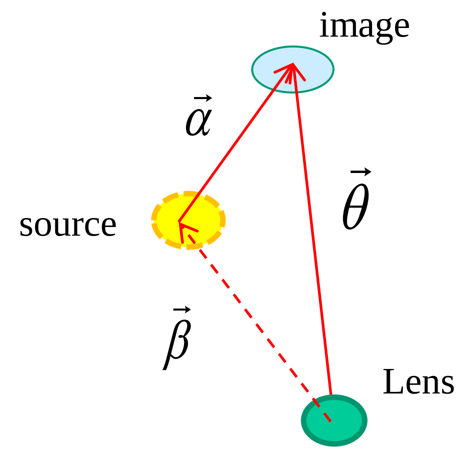
- Depends only on **gravity** (based on the general relativity)
- No need of any assumption on "**dynamical state**" or "**matter content**" of the system
  baryonic matter <=> dark matter
- Complementary to other methods **(X-ray, SZ effect, optical)**

(2) Importance as means of cosmological tests since it depends also on the global geometry of the universe

- Measurement of cosmological parameters
  Dark energy equation of state

Basic of gravitational Lensing





Configuration in sky

$$D_{os}\vec{\theta} = D_{os}\vec{\beta} + D_{ls}\vec{\hat{\alpha}} \quad (\vec{PI} = \vec{PS} + \vec{SI})$$

$$\vec{\theta} - \vec{\beta} = \frac{D_{ls}}{D_s}\vec{\hat{\alpha}} \equiv \vec{\alpha}(\vec{\theta})$$

$D_{ls} = D(z_l, z_s, \Omega_m, \Omega_\Lambda)$ Ang.diam.distance

Lens equation

Bending angle

$$\vec{\alpha} = \frac{2}{c^2}\frac{D_{ls}}{D_s}\int dz \vec{\nabla}_\perp \Phi_N \equiv \vec{\partial}\psi \quad \Rightarrow \quad \partial_\theta \cdot \vec{\alpha} = \Delta_\theta \psi \equiv 2\kappa \quad (\Delta\Phi_N = -4\pi G\rho)$$

$$\kappa(\vec{\theta}) \equiv \frac{\Sigma(\vec{\theta})}{\Sigma_{cr}} \quad \text{with} \quad \Sigma_{cr} = \frac{c^2 D_s}{4\pi G D_l D_{ls}}$$

convergence（normalized surface mass density）

Bending angle

Newtonian metric

$$ds^2 = -\left(1 + \frac{2\Phi}{c^2}\right)c^2 dt^2 + \left(1 - \frac{2\Phi}{c^2}\right)(dx^2 + dy^2 + dz^2)$$

$$\Phi = -G\int \frac{\rho(\vec{r}\,')}{\left|\vec{r} - \vec{r}\,'\right|}\, d^3x' \qquad \text{Newton potential}$$

If light ray propagates along z-direction

$$ds^2 = 0 \quad \Rightarrow \quad v = \frac{dz}{dt} \approx \frac{c}{1 - 2\Phi / c^2} \equiv \frac{c}{n(x)}$$

⇒ Gravitational field has the index of refraction $n = 1 - \frac{2\Phi}{c^2}$

$$\vec{\hat{\alpha}} = -\int dz \vec{\nabla}_{\perp} n = \frac{2}{c^2}\int dz \vec{\nabla}_{\perp}\Phi$$

Lens potential

$$\vec{\alpha}(\vec{\theta}) \equiv \vec{\nabla}_\theta \psi(\vec{\theta}) = \frac{D_{ls}}{D_s} \frac{2}{c^2} \int dz\ \vec{\nabla}_{\vec{y}} \Phi$$

$$\Delta_\theta \psi = \frac{D_{ls} D_l}{D_s} \frac{2}{c^2} \int dz\ \Delta_y \Phi = \frac{D_{ls} D_l}{D_s} \frac{2}{c^2} \int dz\ 4\pi\ G\rho = \frac{D_{ls} D_l}{D_s} \frac{8\pi G}{c^2} \Sigma$$

$$\nabla_\theta \equiv \frac{\partial}{\partial \vec{\theta}} = D_\ell \frac{\partial}{\partial \vec{y}} = D_\ell \nabla_y \qquad (\ \vec{y} = D_l\ \vec{\theta}\ )$$

$$\Sigma(\vec{\theta}) \equiv \int dz\ \rho(D_l \vec{\theta}, z)$$

$$\kappa(\vec{\theta}) \equiv \frac{\Sigma(\vec{\theta})}{\Sigma_{cr}}$$ convergence ➡ $$\boxed{\Delta_\theta \psi = 2\kappa}$$

$$\Sigma_{cr} = \frac{c^2 D_s}{4\pi G D_l D_{ls}} \approx 0.35 \left(\frac{D}{1Gpc} \right)^{-1} g/cm^2$$ Critical surface mass density

$$\boxed{\vec{\alpha}(\vec{\theta}) = \frac{4G}{c^2} \frac{D_{ls}}{D_s} \int d^2\xi' \Sigma(\vec{\xi}') \frac{\vec{\xi} - \vec{\xi}'}{|\vec{\xi} - \vec{\xi}'|^2} = \frac{1}{\pi} \int d^2\theta' \kappa(\theta') \frac{\vec{\theta} - \vec{\theta}'}{|\vec{\theta} - \vec{\theta}'|^2}}$$

Lens mapping

Deformation of the source

$$\vec{\beta} = \vec{\theta} - \vec{\nabla}\psi(\vec{\theta})$$

If the source size is much smaller than the scale of variation of the potential

$$\vec{\beta}(\vec{\theta}) \approx \vec{\beta}(\vec{\theta}_0) + \frac{\partial\vec{\beta}}{\partial\vec{\theta}} \cdot (\vec{\theta} - \vec{\theta}_0)$$

$$A(\vec{\theta}) = \frac{\partial\vec{\beta}}{\partial\vec{\theta}}(\vec{\theta}) = \delta_{ij} - \frac{\partial^2\psi}{\partial\theta_i\partial\theta_j}$$

Jacobean matrix of the lens mapping between the image and source planes

Convergence and shear

$$A(\vec{\theta}) = \delta_{ij} - \frac{\partial^2 \psi}{\partial \theta_i \partial \theta_j} = \begin{pmatrix} 1-\psi_{11} & -\psi_{12} \\ -\psi_{12} & 1-\psi_{22} \end{pmatrix} \qquad \left(\psi_{ij} \equiv \frac{\partial^2 \psi}{\partial \theta_i \partial \theta_j} \right)$$

$$1-\psi_{11} = 1 - \frac{1}{2}(\psi_{11}+\psi_{22}) - \frac{1}{2}(\psi_{11}-\psi_{22})$$

$$A = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix} = (1-\kappa)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & -\gamma_1 \end{pmatrix}$$

convergence

$$\kappa = \frac{1}{2}\Delta_\theta \psi = \frac{1}{2}(\psi_{,11}+\psi_{,22}),$$

(Gravitational) shear

$$\gamma_1 = |\gamma|\cos(2\phi) = \frac{1}{2}(\psi_{,11}-\psi_{,22}),$$

$$\gamma_2 = |\gamma|\sin(2\phi) = \psi_{,12}$$

Relation between convergence and shear

In Fourier Space

$$\psi(\vec{\theta}) = \int d^2k\ \hat{\psi}(k) e^{i\vec{k}\cdot\vec{\theta}}$$

Then the Fourier components of convergence and shear are as follows

$$\hat{\kappa}(k) = -\frac{1}{2}\vec{k}^2 \hat{\psi}(k)$$

$$\hat{\gamma}(k) = -\left[\frac{1}{2}(k_1^2 - k_2^2) + i k_1 k_2\right]\hat{\psi}(k)$$

Therefore

$$\hat{\gamma}(k) = \frac{1}{\pi}\hat{\kappa}(k)\hat{D}(k) \quad \text{with} \quad \hat{D}(k) = \pi\frac{k_1^2 - k_2^2 + 2ik_1k_2}{k^2} = \pi\ e^{2i\phi_k},\ \ \vec{k} = (k, \phi_k)$$

$$\hat{D}(k)\hat{D}^*(k) = \pi^2 \quad \Rightarrow \quad \boxed{\hat{\kappa}(k) = \frac{1}{\pi}\hat{\gamma}(k)\hat{D}^*(k)}$$

In real space

$$\kappa(\vec{\theta}) = \frac{1}{\pi}\int d^2\theta' D^*(\vec{\theta} - \vec{\theta}')\gamma(\vec{\theta}')$$

$$D(\vec{\theta}) = \left[\frac{1}{2}\left(\frac{\partial^2}{\partial\theta_1^2} - \frac{\partial^2}{\partial\theta_2^2}\right) + i\frac{\partial^2}{\partial\theta_1\partial\theta_2}\right] \ell n |\vec{\theta}| = \frac{\theta_2^2 - \theta_1^2 - 2i\theta_1\theta_2}{|\vec{\theta}|^4}$$

Meaning of γ

Diagonarize A

$$\det(A-\lambda I)=0 \;\Rightarrow\; \lambda_{\pm}=1-\kappa\pm|\gamma|=(1-\kappa)(1\pm|g|);\quad |g|=\frac{|\gamma|}{1-\kappa},\quad \lambda_{+}\geq\lambda_{-}(\kappa<1)$$

$$A\vec{V}_{\pm}=\lambda_{\pm}\vec{V}_{\pm}$$

$$\vec{V}_{+}=\begin{pmatrix}\sin\phi\\ -\cos\phi\end{pmatrix}=\begin{pmatrix}\cos(\phi-\pi/2)\\ \sin(\phi-\pi/2)\end{pmatrix},\quad \vec{V}_{-}=\begin{pmatrix}\cos\phi\\ \sin\phi\end{pmatrix},\quad \phi=\frac{1}{2}\tan^{-1}\left(\frac{\gamma_2}{\gamma_1}\right)$$

$$A_{ij}'(\vec{\theta}_0)=O_{ik}(\phi)O_{jl}(\phi)A_{kl}(\vec{\theta}_0)=\begin{pmatrix}\lambda_{-} & 0\\ 0 & \lambda_{+}\end{pmatrix}\qquad O\equiv\begin{pmatrix}\cos\phi & -\sin\phi\\ \sin\phi & \cos\phi\end{pmatrix}$$

$$\delta\beta=A(\theta_0)\;\delta\theta\qquad \begin{pmatrix}\delta\beta_1\\ \delta\beta_2\end{pmatrix}=\begin{pmatrix}1-\kappa-\gamma_1 & -\gamma_2\\ -\gamma_2 & 1-\kappa+\gamma_1\end{pmatrix}\begin{pmatrix}\delta\theta_1\\ \delta\theta_2\end{pmatrix}$$

$$\underbrace{O\delta\beta}_{\delta\beta'}=\underbrace{O\;A(\theta_0)O^T}_{A'(\theta_0)}\underbrace{O\delta\theta}_{\delta\theta'} \;\Rightarrow\; \delta\beta'=A'(\theta_0)\;\delta\theta'$$

Consider an unit circle in the source plane

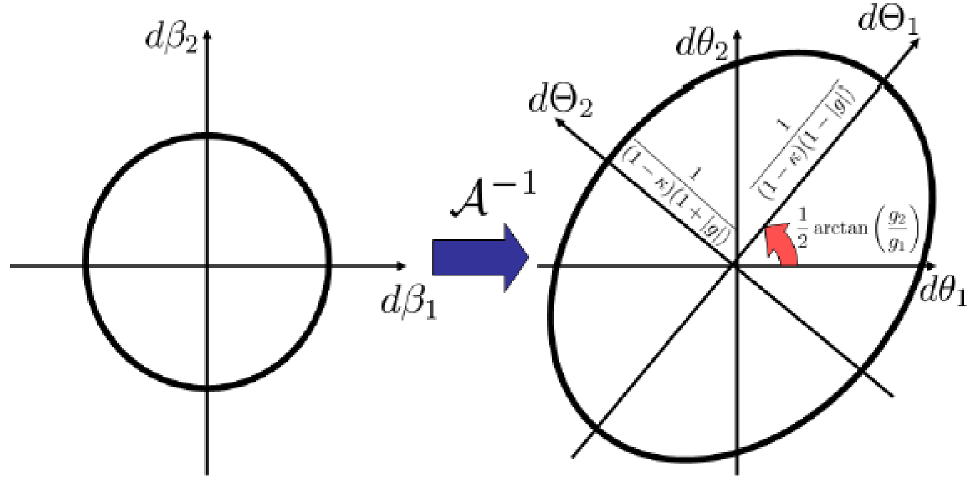
$$\delta\beta^T \delta\beta = 1 \quad \Rightarrow \quad d\theta'^T A'^T A' \; d\theta' = 1$$

$$\begin{pmatrix} \delta\theta_1' & \delta\theta_2' \end{pmatrix} \begin{pmatrix} \lambda_+ & 0 \\ 0 & \lambda_- \end{pmatrix} \begin{pmatrix} \lambda_+ & 0 \\ 0 & \lambda_- \end{pmatrix} \begin{pmatrix} \delta\theta_1' \\ \delta\theta_2' \end{pmatrix} = 1 \quad \Rightarrow \quad \frac{\delta\theta_1'^2}{1/\lambda_+^2} + \frac{\delta\theta_2'^2}{1/\lambda_-^2} = 1$$

A circle in the source plane is mapped to an ellipse with major axis $1/\lambda-$ and minor axis $1/\lambda+$ ($\kappa<1$)

Major axis is directed from $d\theta 1$ axis by angle

$$\phi = \frac{1}{2}\tan^{-1}\left(\frac{\gamma_2}{\gamma_1}\right)$$



magnification

Since the surface brightness is conserved in the lens mapping, the magnification is given by the ratio of area between source and image

$$\mu(\vec{\theta}) = \det\left(\frac{\partial\vec{\theta}}{\partial\vec{\beta}}\right) = \frac{1}{\det A} = \frac{1}{\lambda_+\lambda_-} = \frac{1}{(1-\kappa)^2 - |\gamma|^2}$$

Critical curves and caustics

The closed curves in the image plane defined by

$$\det A(\vec{\theta}) = 0$$

or, equivalently

$$0 = \lambda_+ = 1 - \kappa + |\gamma|,$$ Inner (radial) critical curve

$$0 = \lambda_- = 1 - \kappa - |\gamma|$$ outer (tangential) critical curve

are called **critical** curves on which the magnification factor diverges, and those mapped into the source plane are called **caustics**

3.2 Some examples

- Circular symmetric lens
- Elliptical lens

Circular symmetric lens

$$\vec{\alpha}(\vec{\theta}) = \frac{4G}{c^2}\frac{D_{ls}}{D_s D_l}\int d^2\theta' \Sigma(\vec{\theta}')\frac{\vec{\theta}-\vec{\theta}'}{|\vec{\theta}-\vec{\theta}'|^2}$$

Circular symmetry $\Sigma(\vec{\theta}) = \Sigma(\theta) : \theta = |\vec{\theta}|$

$$\vec{\alpha}(\theta) = \frac{4G}{c^2}\frac{D_{ls}}{D_s D_l}\frac{\vec{\theta}}{\theta^2} M(<\theta)$$

where

$$M(<\theta) \equiv 2\pi\int_0^\theta d\theta' \theta' \Sigma(\theta')$$

proof

$$\vec{I} \equiv \int_0^{2\pi} d\phi \frac{\vec{\theta}-\vec{\theta}'}{|\vec{\theta}-\vec{\theta}'|^2}$$

$$\theta = \theta_x + i\theta_y$$

$$I \equiv \int_0^{2\pi} d\phi \frac{\theta-\theta'}{|\theta-\theta'|^2} = \int_0^{2\pi} d\phi \frac{1}{\theta^* - \theta'^*}$$

$$\theta' = |\theta'| e^{i\phi}$$

$$I = \int_0^{2\pi} d\phi \frac{1}{\theta^* - |\theta'| e^{-i\phi}} = \oint_{|\xi|=1} \frac{d\xi}{i\xi} \frac{1}{\theta^* - |\theta'| \xi^{-1}} \qquad (\xi = e^{i\phi})$$

$$= \frac{1}{i\theta^*} \oint_{|\xi|=1} \frac{d\xi}{\xi - |\theta'|/\theta^*} = \frac{2\pi}{\theta^*} \Theta(|\theta| - |\theta'|)$$

Step function

$$= \frac{2\pi}{|\theta|^2} \theta \, \Theta(|\theta| - |\theta'|)$$

$$\int d^2\theta' \Sigma(\vec{\theta}') \frac{\vec{\theta}-\vec{\theta}'}{|\vec{\theta}-\vec{\theta}'|^2} = \int d\theta' \theta' \Sigma(\theta') \int_0^{2\pi} d\phi \frac{\vec{\theta}-\vec{\theta}'}{|\vec{\theta}-\vec{\theta}'|^2}$$

$$= \int d\theta' \theta' \Sigma(\theta') \frac{2\pi}{|\theta|^2} \vec{\theta} \, \Theta(|\vec{\theta}| - |\vec{\theta}'|) = \frac{2\pi}{|\theta|^2} \vec{\theta} \int_0^{\theta} d\theta' \, \theta' \Sigma(\theta')$$

Einstein ring

$$\vec{\alpha}(\vec{\theta}) = \frac{D_{ls}}{D_l D_s} \frac{4GM(\theta)}{c^2 \theta^2} \vec{\theta}$$

$M(\theta)$ Projected mass enclosed by a circle with angular radius θ

Einstein ring $\beta = 0$

$$0 = \theta - \frac{D_{ls}}{D_l D_s} \frac{4GM(\theta)}{c^2 \theta} = \theta - \frac{1}{\pi \Sigma_{cr}} \frac{M(\theta)}{\theta} \Rightarrow \theta_E = \left[\frac{M(\theta_E)}{\pi \Sigma_{cr}} \right]^{1/2}$$

Mass estimate using luminous arc

$$< \Sigma(\theta_E) >= \frac{1}{\pi \theta_E^2} \int_0^{\theta_E} d\theta \theta \int_0^{2\pi} d\varphi \, \Sigma(\theta) = \Sigma_{cr}$$

$$\theta = \theta_{arc} \approx \theta_E$$

$$M(\theta_{arc}) = \Sigma_{cr} \pi (D_l \, \theta_{arc})^2$$

$$\sim 1.1 \times 10^{15} M_{solar} \left(\frac{\theta_{arc}}{30''} \right)^2 \left(\frac{D}{1Gpc} \right)$$

Point mass

$$M(\theta) = M = const.$$

$$\beta = \theta - \frac{4GM}{c^2}\frac{D_{ls}}{D_s D_l}\frac{1}{\theta} = \theta - \frac{\theta_E^2}{\theta} \quad \Rightarrow \quad \psi = \theta_E^2 \ell n\,|\,\theta\,|$$

Einstein angle

$$\theta_E = \left[\frac{4GM}{c^2}\frac{D_{ls}}{D_s D_l}\right]^{1/2} \sim 1''\left(\frac{M}{10^{11} M_{solar}}\right)^{1/2}\left(\frac{D}{1Gpc}\right)^{-1/2}$$

$$\sim 1\ mas\ \left(\frac{M}{M_{solar}}\right)^{1/2}\left(\frac{D}{10kpc}\right)^{-1/2}$$

Image positions

$$\theta_\pm = \frac{1}{2}\left(\beta \pm \sqrt{\beta^2 + 4\theta_E^2}\right)$$

Distortion and magnification

$$\delta\beta = \delta\theta + \frac{\theta_E^2}{\theta^2}\delta\theta$$

$$W = \frac{\delta\theta}{\delta\beta} = \frac{1}{1+(\theta_E/\theta)^2} < 1$$

Radial distortion

$$L = \frac{\theta}{\beta} = \frac{1}{1-(\theta_E/\theta)^2}$$

Tangential distortion

magnification

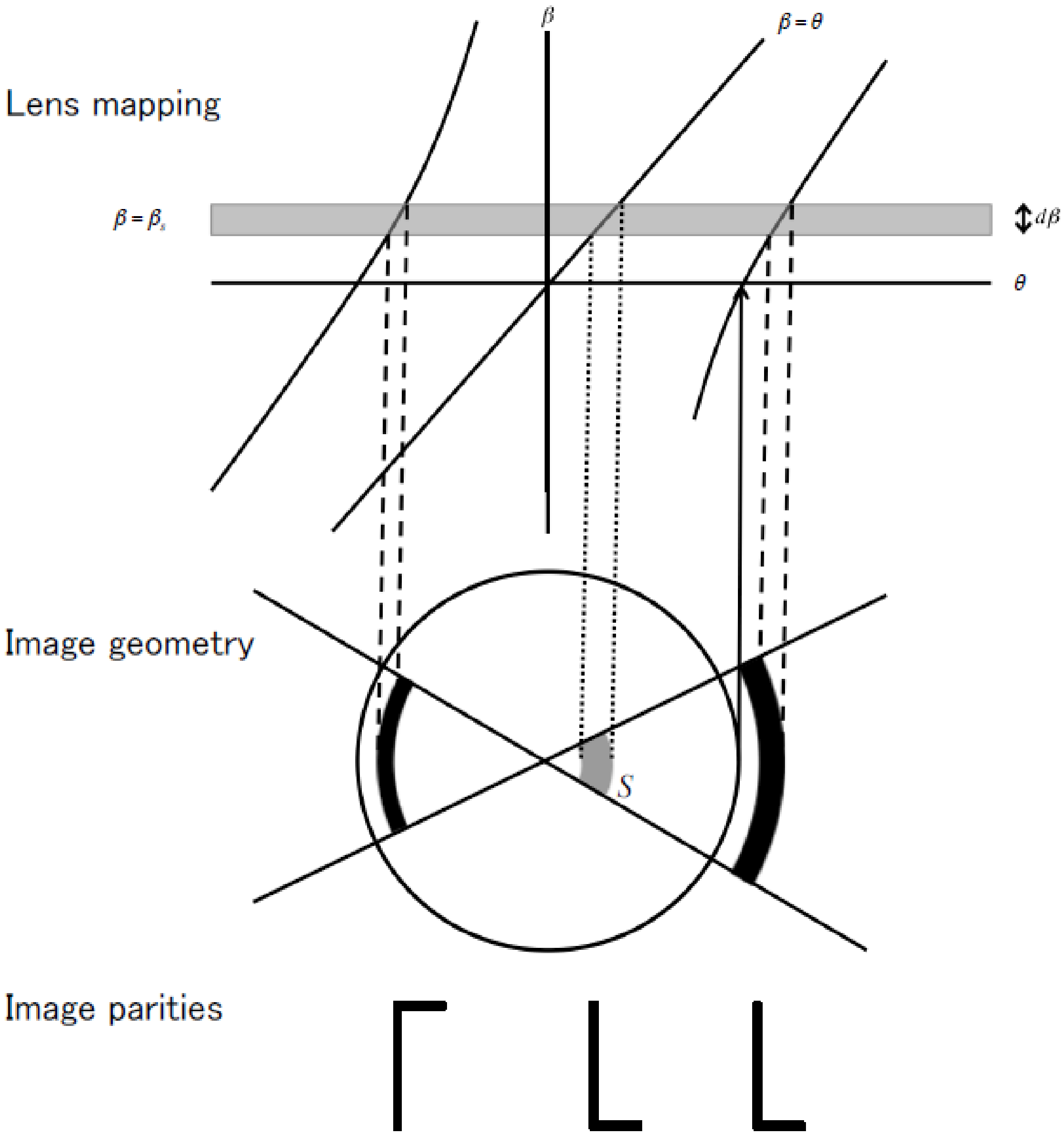
κ

$$\mu_\pm = \frac{\theta\delta\theta}{\beta\delta\beta} = \frac{1}{1-(\theta_E/\theta_\pm)^4} = \frac{u^2+2}{2u\sqrt{u^2+4}} \pm \frac{1}{2}, \qquad (u = \frac{\beta}{\theta_E})$$

Total magnification

$$\mu = \mu_+ + \mu_- = \frac{u^2+2}{u\sqrt{u^2+4}}$$

$$\mu = 1.34 \quad for \quad u = 1 \Rightarrow \Delta m = 0.34$$



Lens mapping
β
β = θ
β = βs
dβ
θ
Image geometry
S
Image parities

Singular Isothermal Sphere(SIS)

The equation of state for a gas composed of stars

$$p = \frac{\rho\, kT}{m}$$

Isothermal means $m\sigma^2 = kT \;\Rightarrow\; p = \sigma^2 \rho$

$$\frac{1}{\rho}\frac{dp}{dr} = g$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 g\right) = -4\pi G\rho$$

$$\Rightarrow \quad \frac{\sigma^2}{r^3}\frac{d}{dr}\left(\frac{r^3}{\rho}\frac{d\rho}{dr}\right) = -4\pi G\rho$$

$$\Rightarrow \quad \rho_{SIS} = \frac{\sigma^2}{2\pi G}\frac{1}{r^2}$$

Surface mass density

$$\Sigma = \int dz\ \rho = \int_{-\infty}^{+\infty} \frac{dz}{R^2 + z^2} = \frac{\sigma^2}{2G}\frac{1}{R}, \qquad R = \sqrt{x^2 + y^2}$$

2D mass within an angular radius θ

$$M(<\theta) = 2\pi \int_0^{\xi} d\xi\ \xi\ \Sigma(\xi) = \frac{\pi\sigma^2}{G}\xi = \frac{\pi\sigma^2}{G} D_l \theta, \qquad \xi = D_\ell \theta$$

Bending angle

$$\vec{\hat{\alpha}} = 4GD_\ell M(<\theta)\frac{\vec{\theta}}{\theta^2} = 4\pi\left(\frac{\sigma}{c}\right)^2 \frac{\vec{\theta}}{\theta}$$

$$\left|\vec{\hat{\alpha}}\right| \sim 1.4''\left(\frac{\sigma}{220\,\mathrm{km/s}}\right)^2 \sim 2.6''\left(\frac{\sigma}{300\,\mathrm{km/s}}\right)^2$$

Lens equation

$$\beta = \theta - \frac{D_{ls}}{D_s}\hat{\alpha} = \theta - \frac{D_{ls}}{D_s}4\pi\left(\frac{\sigma}{c}\right)^2 = \theta - \theta_E \qquad \left(\theta_E = \frac{D_{ls}}{D_s}4\pi\left(\frac{\sigma}{c}\right)^2\right)$$

Lens potential

$$\psi_{SIS}(\vec{\theta}) = \theta_E \theta$$

Convergence and shear

$$\kappa(\theta) = |\gamma(\theta)| = \frac{\theta_E}{2\theta}$$

Lens Image by SIS lens

$$\beta = \theta - \theta_E \qquad \theta_E \equiv \frac{D_{ls}}{D_s} 4\pi \left(\frac{\sigma}{c} \right)^2$$

Image position

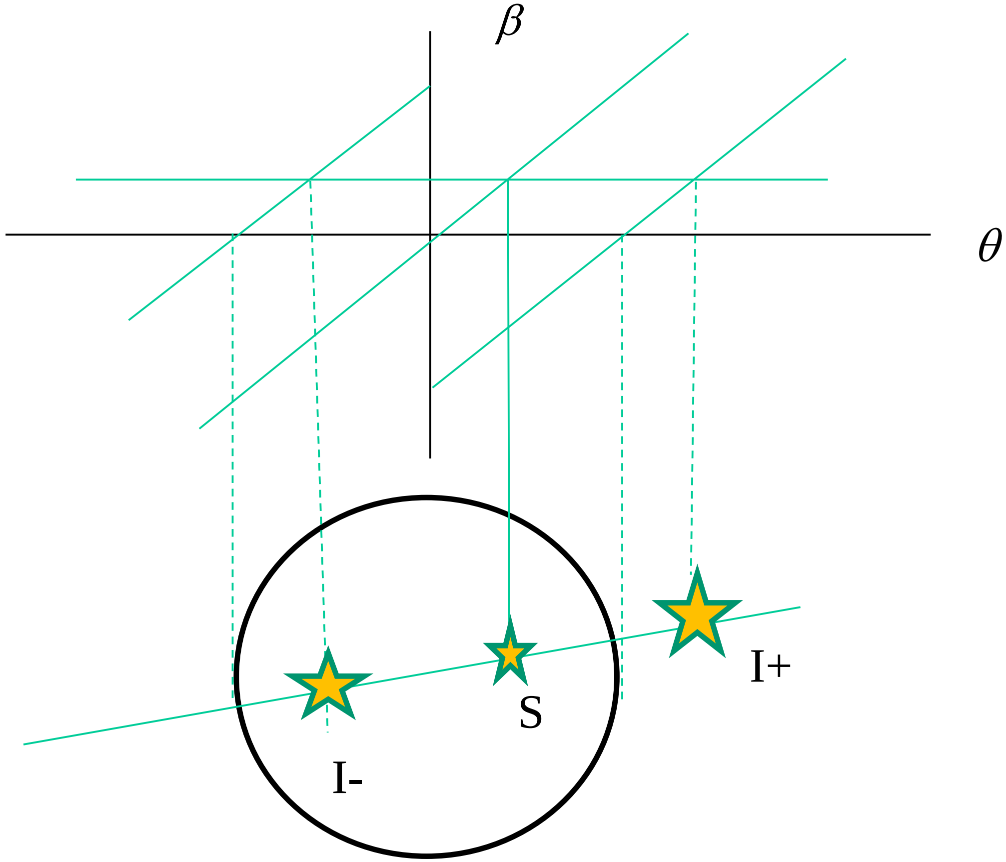
$$|\beta| \le \theta_E \;\Rightarrow 2 \text{ images at } \theta_{I\pm} = \theta_E \pm \beta$$

$$|\beta| > \theta_E \;\Rightarrow 1 \text{ image at } \theta_I = \theta_E + \beta$$

$$L = \frac{\theta}{\beta} = \frac{1}{1 - \theta_E / \theta}$$

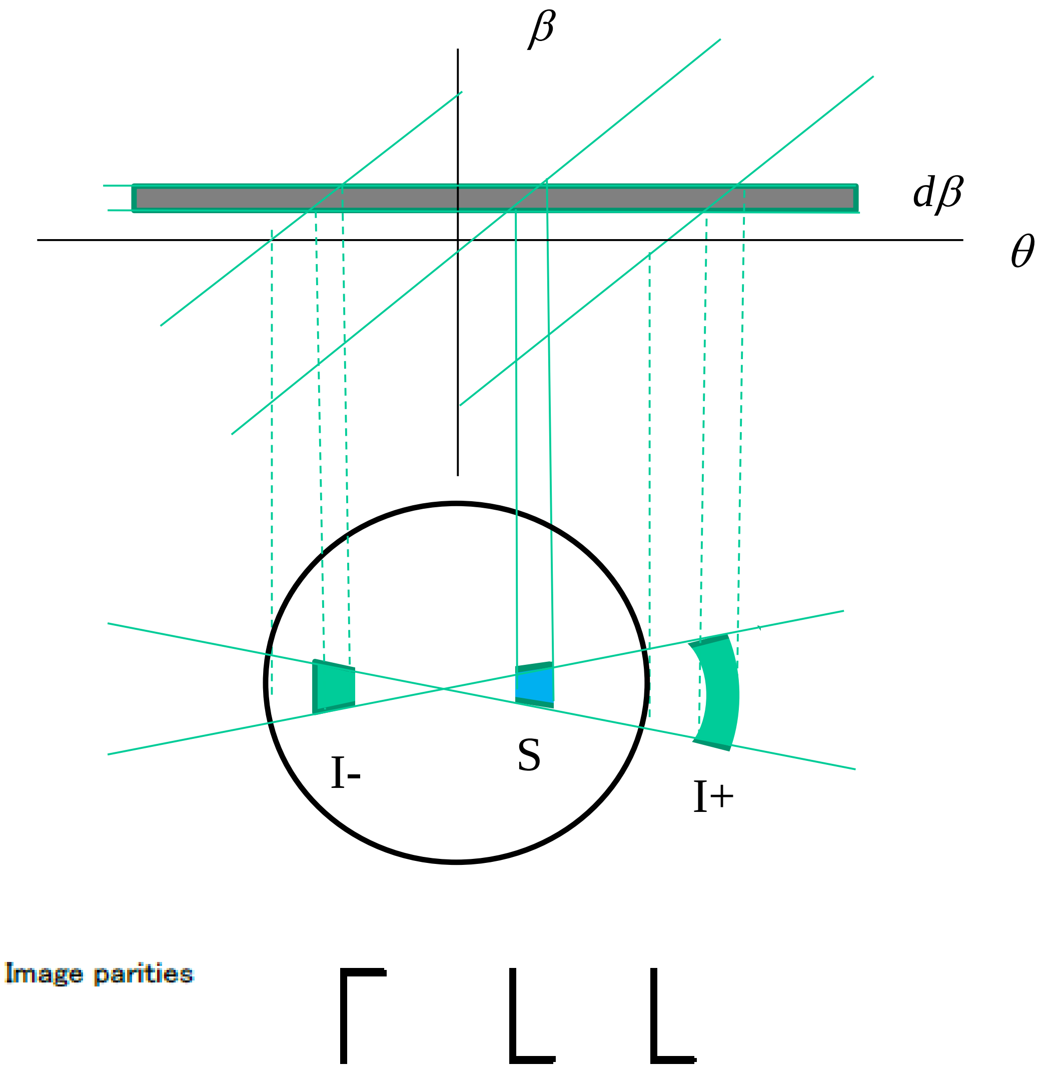
$$W = \frac{d\theta}{d\beta} = 1$$

$$\mu = \frac{\theta d\theta}{\beta d\beta} = \frac{1}{1 - \theta_E / \theta} = \begin{cases} \dfrac{\theta_E \pm \beta}{\beta} & \text{for 2 images } 2 \le \mu_+ < \infty \ (\beta > 0) \\ \dfrac{\theta_E}{\beta} + 1 & \text{for 1 image } 1 \le \mu < 2 \end{cases}$$



β
θ
I-
S
I+

Lens mapping of SIS



SIS with a finite core(CIS)

$$\rho_{CIS} = \frac{\sigma^2}{2\pi G}\frac{1}{r^2 + r_c^2} = \frac{\rho_c}{1 + r^2 / r_c^2}, \qquad \rho_c \equiv \frac{\sigma^2}{2\pi G}\frac{1}{r_c^2}$$

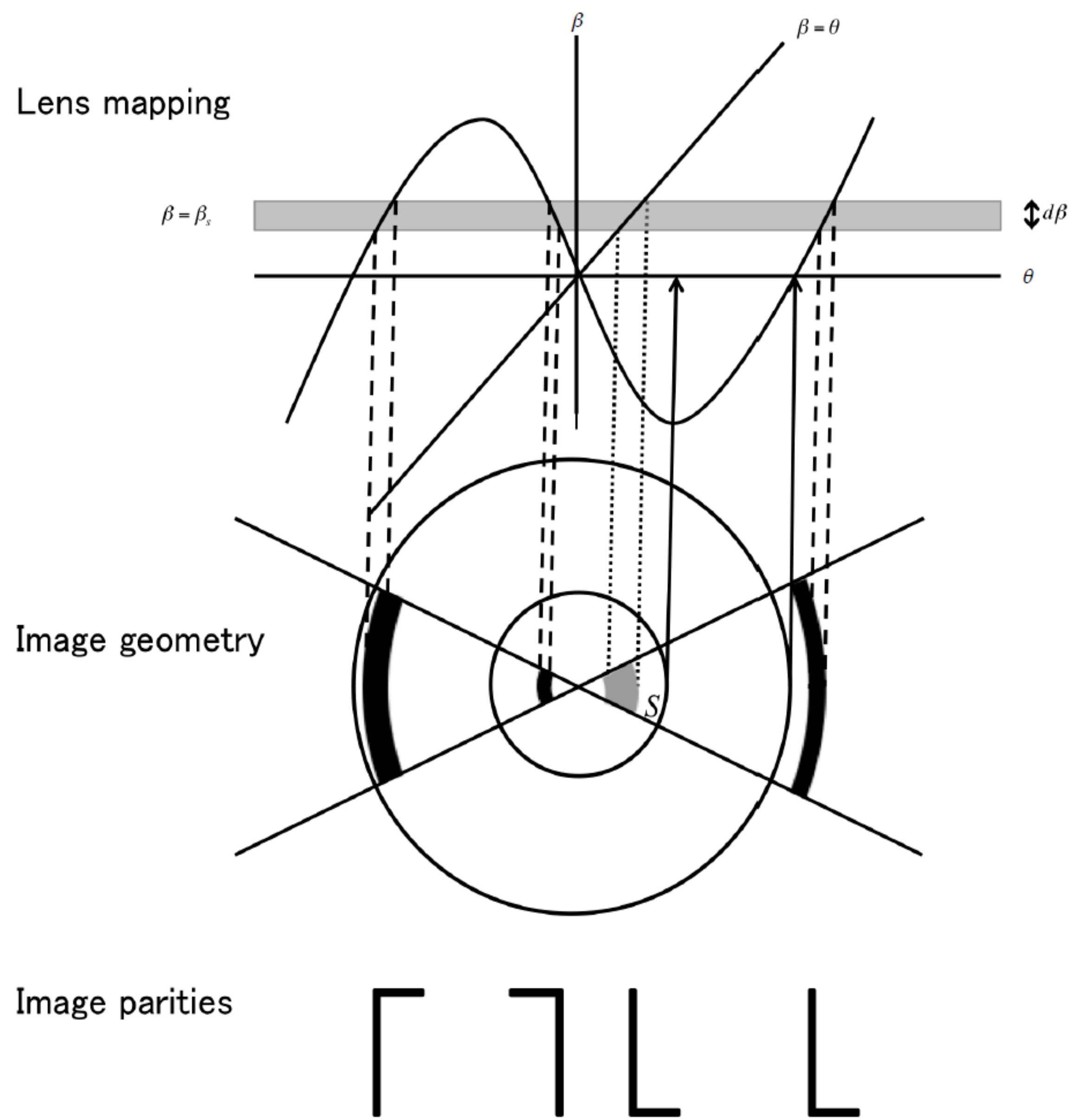
$$\Sigma = \int dz\ \rho = \frac{\pi\rho_c}{\sqrt{1 + R^2 / r_c^2}} = \frac{\sigma^2}{2G\, r_c}\frac{1}{\sqrt{1 + R^2 / r_c^2}} \qquad R = \sqrt{x^2 + y^2}$$

$$= \frac{\sigma^2}{2G}\frac{1}{D_L\theta_c}\frac{1}{\sqrt{1 + \theta^2 / \theta_c^2}} \qquad \theta_{\mathrm{c}} \equiv r_c / D_L$$

$$M(<\theta) = \frac{\pi\sigma^2}{G}\frac{1}{D_L}\left(\sqrt{\theta^2 + \theta_c^2} - \theta_c\right)$$

$$\vec{\hat{\alpha}} = 4GD_\ell M(<\theta)\frac{\vec{\theta}}{\theta^2} = 4\pi\left(\frac{\sigma}{c}\right)^2 \frac{\vec{\theta}}{\theta^2}\left(\sqrt{\theta^2 + \theta_c^2} - \theta_c\right)$$

Lens mapping of general spherical lens



NFW model

$$\rho_{\mathrm{NFW}}(r) = \frac{\rho_s}{(r/r_s)(1+r/r_s)^2}$$

$$\frac{d\ln\rho(r)}{d\ln r} = -2 \quad \text{at} \quad r = r_s$$

Concentration parameter

$$c_{200} \equiv \frac{r_{200}}{r_s}, \qquad \rho(r_{200}) = 200\rho_{cr}(z)$$

Surface mass density

$$\Sigma(x) = \int_{-\infty}^{\infty} \rho(r_s x, z)dz = 2\rho_c r_s F(x) \qquad x = \frac{r}{r_s}$$

$$F(x) = \begin{cases} \frac{1}{x^2-1}\left(1 - \frac{1}{\sqrt{1-x^2}}\mathrm{arcch}\frac{1}{x}\right) & (x<1) \\ \frac{1}{3} & (x=1) \\ \frac{1}{x^2-1}\left(1 - \frac{1}{\sqrt{x^2-1}}\mathrm{arcccos}\frac{1}{x}\right) & (x>1) \end{cases}$$

2D mass

$$M(<\theta) = 2\pi \int_0^{\xi} d\xi\ \xi\ \Sigma(\xi) = 4\pi \rho_c r_c g(\theta),$$

where

$$g(x) = \begin{cases} \ln\frac{1}{2} + \frac{1}{\sqrt{1-x^2}} \mathrm{arcch}\frac{1}{x} & (x<1) \\ 1 + \ln\frac{1}{2} & (x=1) \\ \ln\frac{1}{2} + \frac{1}{\sqrt{x^2-1}} \mathrm{arcccos}\frac{1}{x} & (x>1) \end{cases}$$

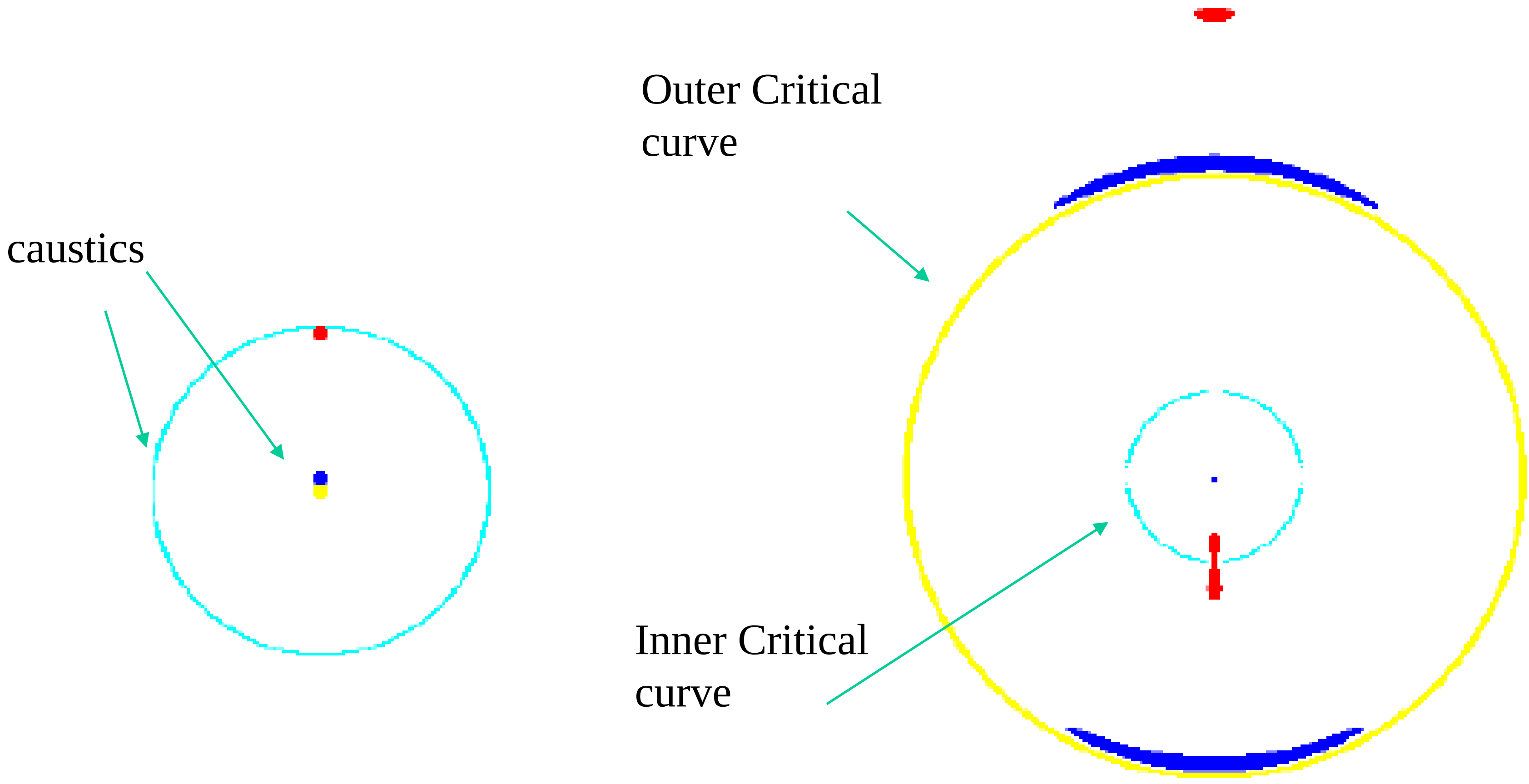
$$\vec{\hat{\alpha}}_{NFW}(x) = 4\kappa_s \theta_s^2 \frac{\vec{\theta}}{\theta} g(x)$$

$$\kappa(x) = 2\kappa_s F(x)$$

$$\gamma(x) = 2\kappa_s \left(\frac{2g(x)}{x^2} - F(x) \right)$$

$$\kappa_s = 2\delta_c \rho_{\mathrm{crit}} r_s / \Sigma_{\mathrm{crit}}$$

Tangential and Radial Arcs



Source plane

Image plane

Example of radial arc:MS2137-23 at z=0.313

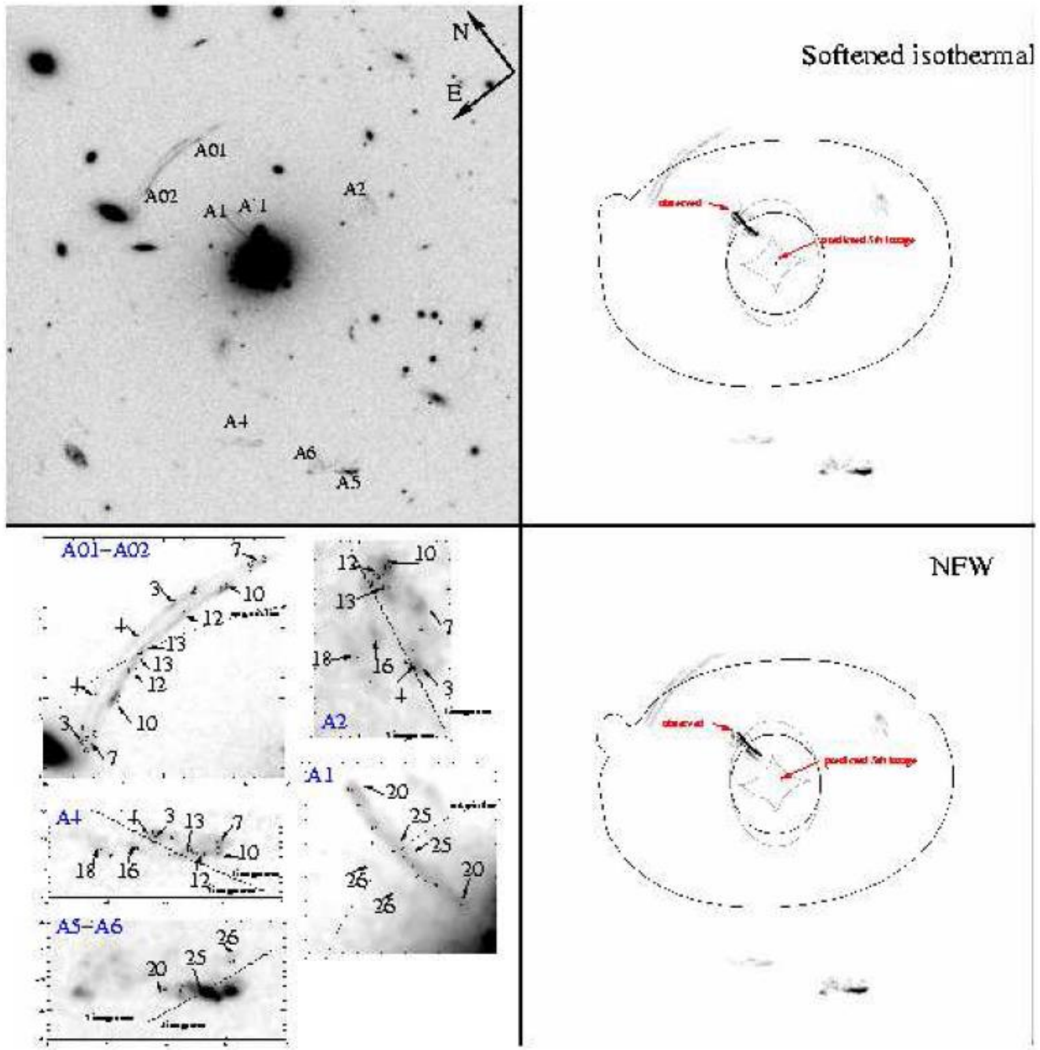


Fig. 1. Upper left panel : overview of the lens configuration. The three arcs systems {A01,A02,A2,A4}, {A1,A5} and {A'1,A6}. The central cD galaxy. This F702 HST field is 56 × 56 arcsec wide (*i.e.* 180 × 180 h^{-1} kpc). Upper (resp. lower) right panel : reconstruction of arcs deduced from the single component best fit IS (resp. NFW) model (see 3.2). In these panels are reported the observed radial arc location. The small azimuthal offset is discussed in Sect. 4.2. The fifth demagnified image predicted by the models near the center is detailed in Fig 4. Lower left panel, detail of some dots used for the model fitting (see Table A.1).

Elliptical lens

CIE (Cored Isothermal Ellipsoid) model

$$\psi(\vec{\theta}) = 4\pi\left(\frac{\sigma}{c}\right)^2 \frac{D_{ls}}{D_s}\sqrt{\theta_c^2 + (1-\varepsilon)\theta_1^2 + (1-\varepsilon)\theta_2^2}$$

σ velocity dispersion

ε Ellipticity: axis ratio $\sqrt{\frac{1-\varepsilon}{1+\varepsilon}}$

Tilted Plummer Elliptical model

$$\psi(\vec{\theta}) = \frac{\alpha_E^2}{\eta}\left(\frac{\theta_c^2 + (1-\varepsilon)\theta_1^2 + (1-\varepsilon)\theta_2^2}{\alpha_E^2}\right)^{\eta/2}$$

Caustics and critical curves of an elliptic lens

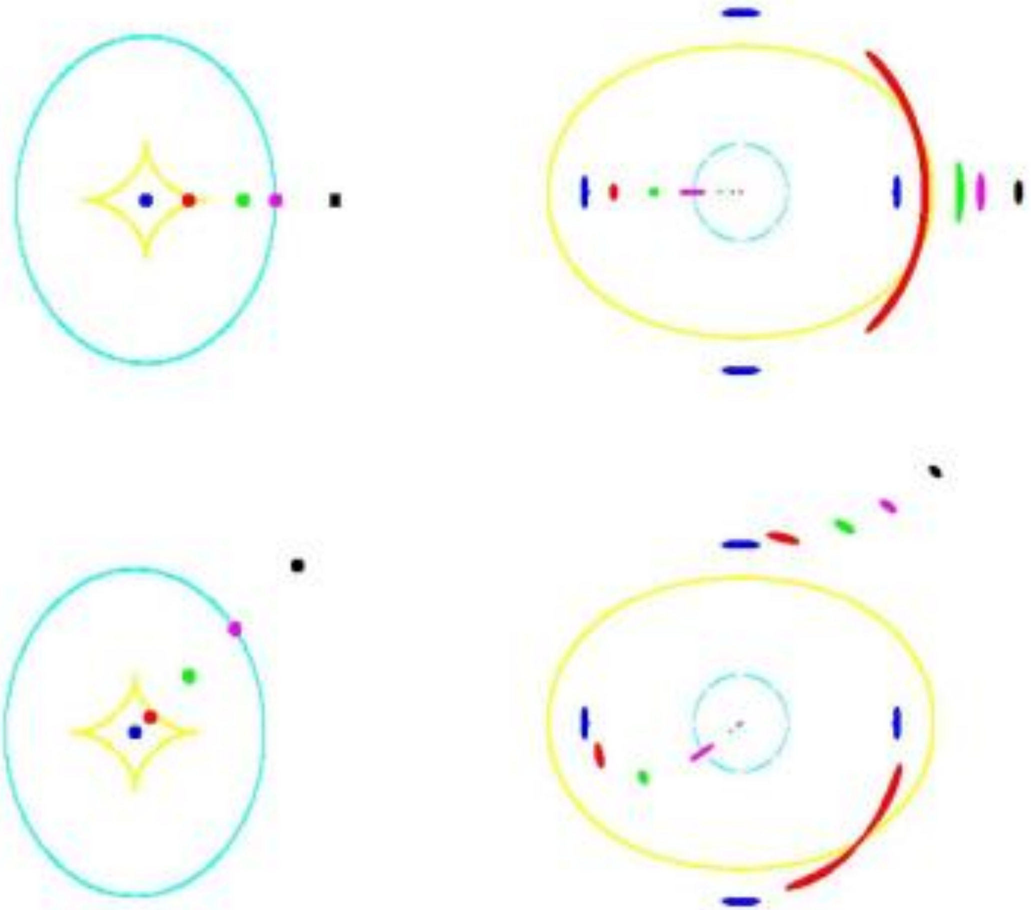
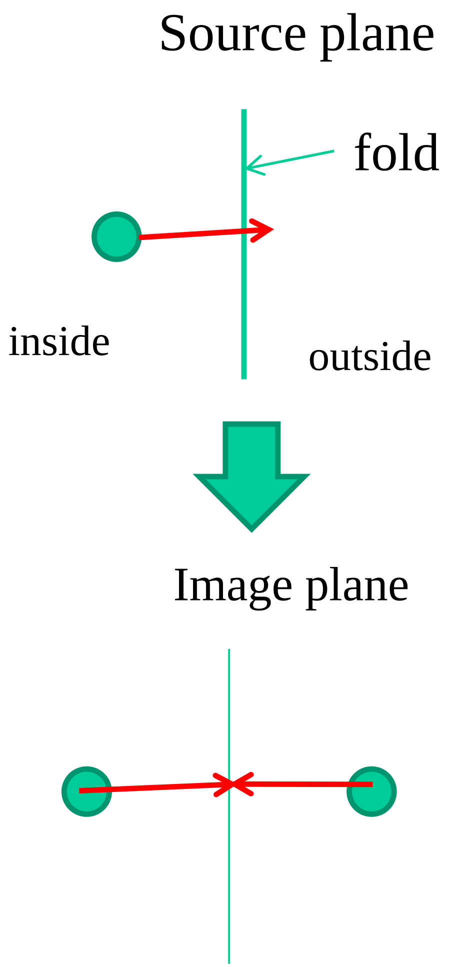
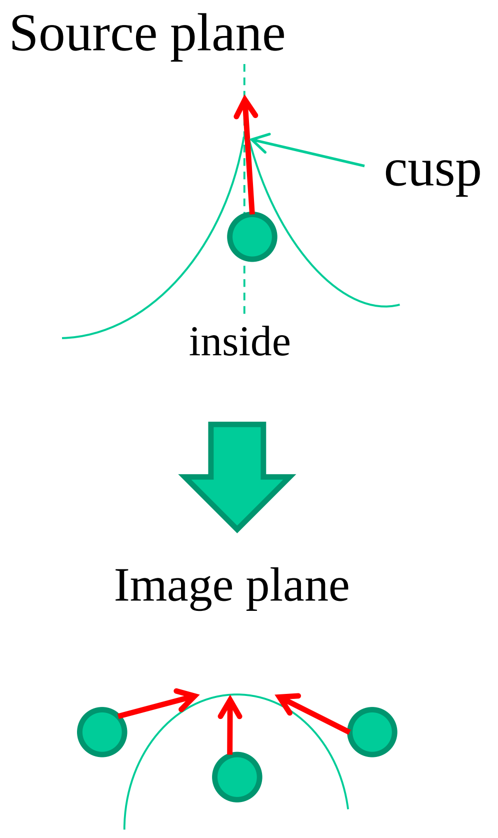
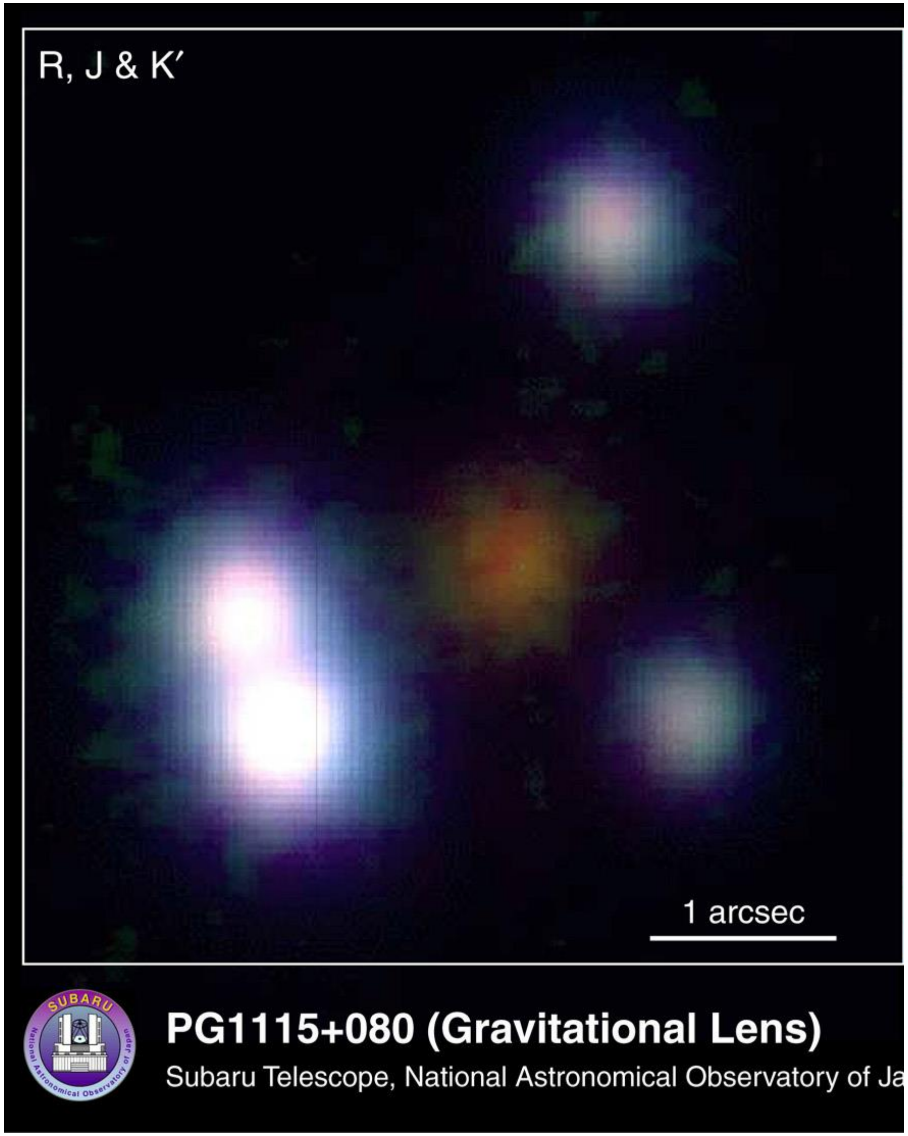


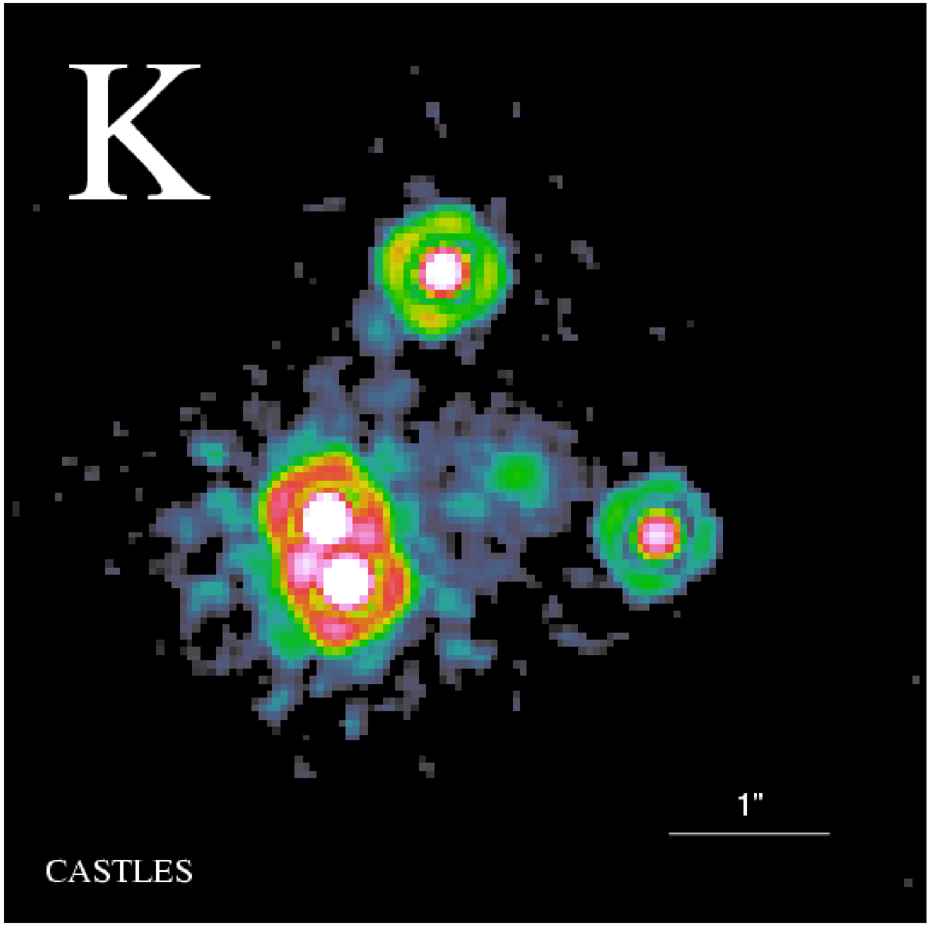
図 2.13　楕円レンズの caustics（左）と critical curves（右）（Hattori, M., Kneib, J.-P., & Makino, N. 1999, *Prog. Theor. Phys. Suppl.*, 133, 1 より転載）.

Fold and cusp

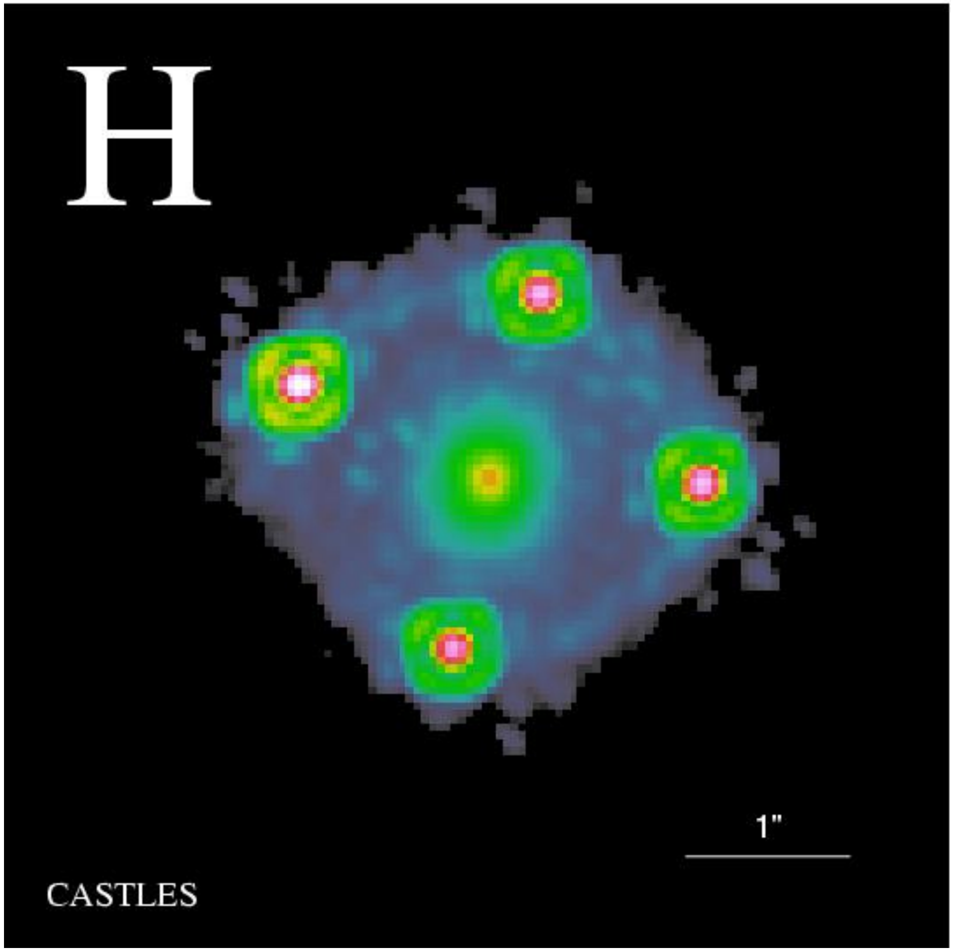


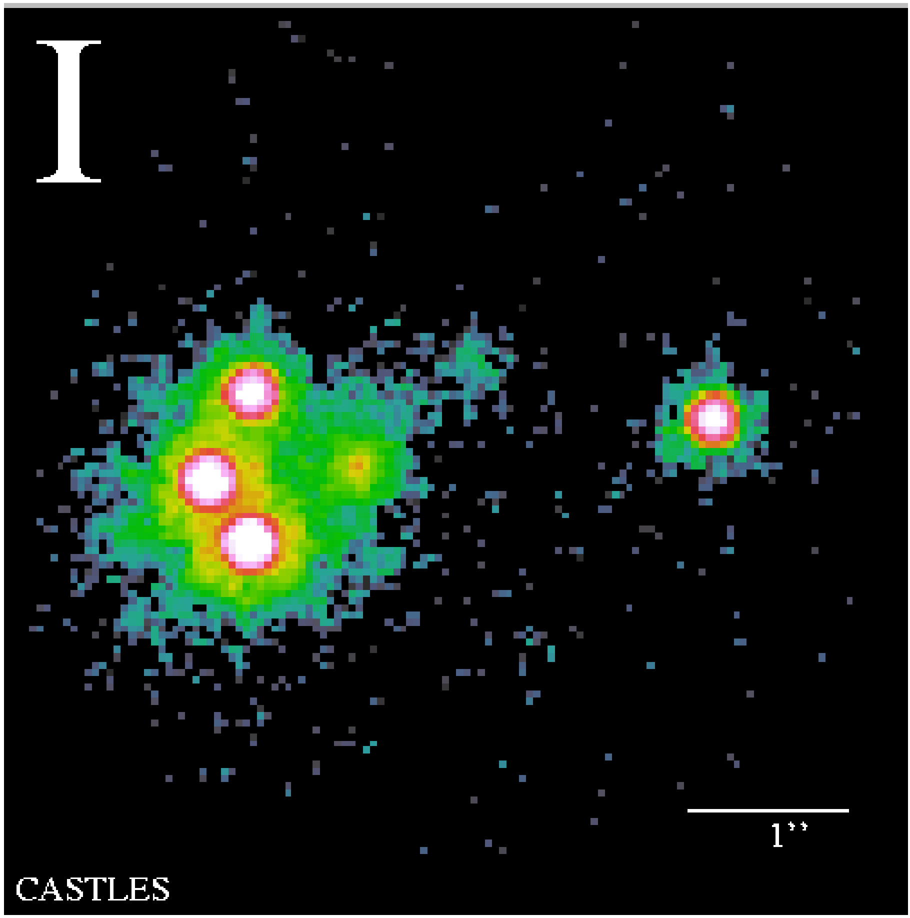



R, J & K′
1 arcsec
PG1115+080 (Gravitational Lens)
Subaru Telescope, National Astronomical Observatory of Ja


K
1"
CASTLES

MG0414+0534





HE0435-1223

RXJ0911+0551

3.3 Strong lensing mass reconstruction

Observables

- Image positions $\vec{\theta}_A : \quad A = 1,\ldots,N$
- Flux ratio between images M_A / M_B
- Time delay $t_{AB} = t_B - t_A$

One choose the lens potential(s) with parameters and then choose source position and the parameters to make χ^2 minimam

$$\chi^2 \equiv \sum_{\mathrm{i}} \frac{\left|\text{predicted value}_{\mathrm{i}} - \text{observed value}_{\mathrm{i}}\right|}{\sigma_i^2}$$

Flux ratio

Magnification matrix is the inverse of Jacobian

$$M = A^{-1}$$

If we have two images A and B

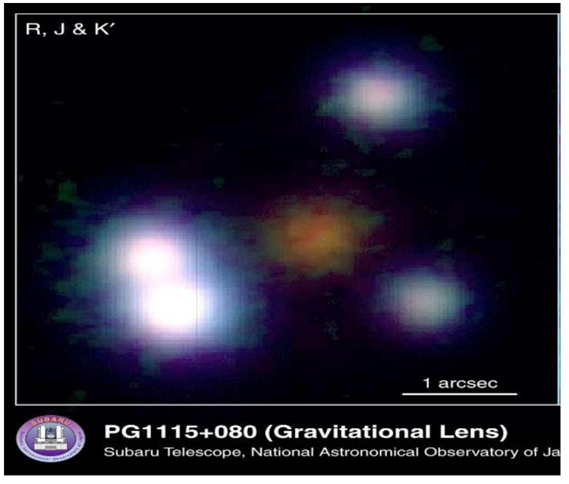
$$\delta\vec{\theta}_B = M(\vec{\theta}_B)_{ij}\delta\beta = M(\vec{\theta}_B)M^{-1}(\vec{\theta}_A)\delta\vec{\theta}_A \equiv M_{BA}\delta\vec{\theta}_B$$

Flux anomaly

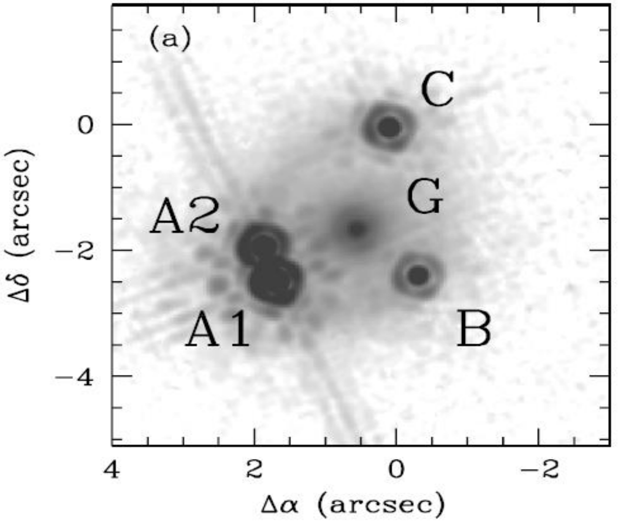
In some lens systems, any smooth potential cannot explain the observed image position and flux ratio between images simultaneously

Example of flux anomaly: PG1115+080 $(z_S = 1.72,\ z_L = 0.31)$

Subaru



HST



Smooth Lens model predicts that A2/A1=1

However the observation shows

A2/A1=0.65±0.02

Impey et al.1998

Time delay

When the source changes its luminosity suddenly at a time, the apparent luminosity of an image will change after some time

The difference of arrival time between the case with lens and without lens is given by

$$\tau = \frac{1+z_L}{H_0}\frac{d_L d_S}{d_{LS}}\left[\frac{1}{2}(\theta-\beta)^2 - \psi(\theta)\right]$$

Geometrical difference

Gravitational time delay

The difference of arrival time between two images

$$\begin{aligned}\Delta_{AB} &= \tau_A - \tau_B \\ &= \frac{1+z_L}{H_0}\frac{d_L d_S}{d_{LS}}\left[\frac{1}{2}\left((\theta_A-\beta)^2 - (\theta_B-\beta)^2\right) - (\psi(\theta_A) - \psi(\theta_B))\right]\end{aligned}$$

In many cases the lensing galaxy is a member of group or cluster. These effects are taken into account by external potential

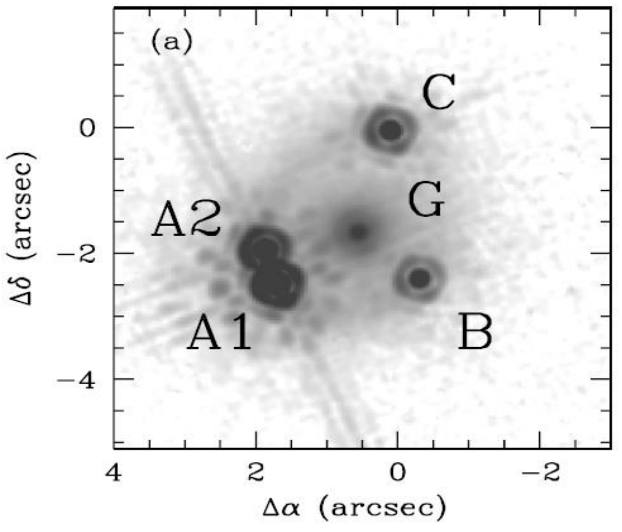
$$\psi_{ext}(\vec{\theta}) = \frac{1}{2}\gamma_1\left(\theta_1^2 - \theta_1^2\right) + \gamma_2\,\theta_1\,\theta_2$$

Since time delay depends on Hubble parameter, it has been used to measure global Hubble parameter

However, there is an ambiguity associated with uniform density sheet

$$H_0 = 100h\,(1 - \kappa_c(0))\ [\mathrm{km/s/Mpc}]$$

Example of time delay: PG1115+080

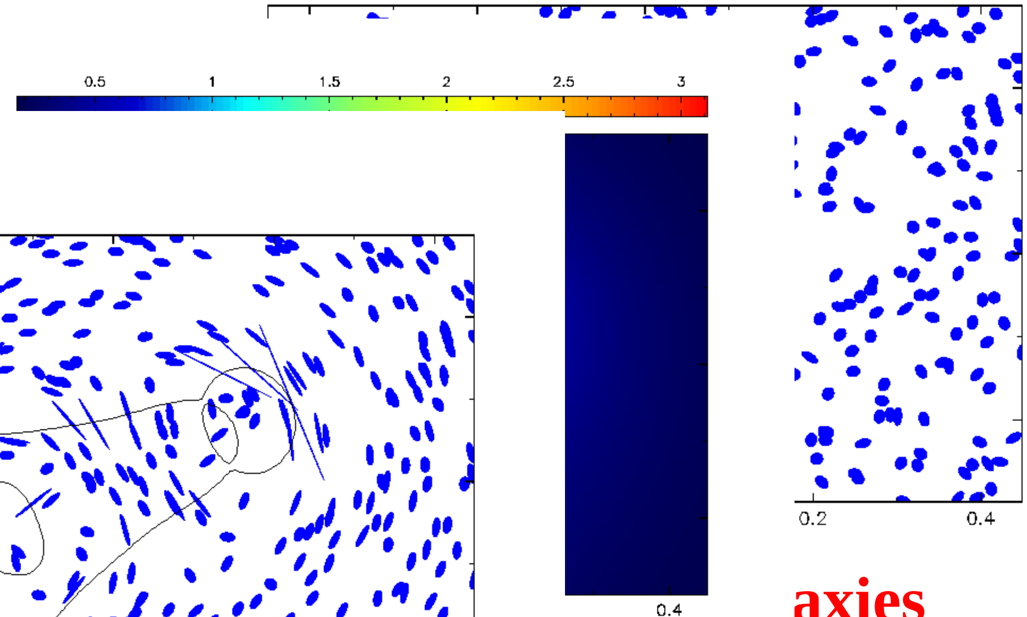


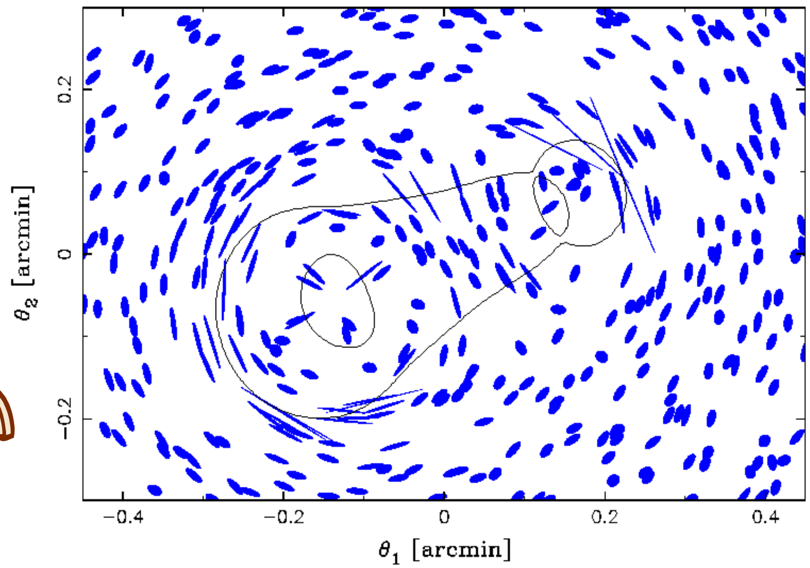
$$\Delta\tau_{AC} = 25.0^{+3.3}_{-3.8} \text{ days}$$

$$r_{ABC} = \frac{\Delta\tau_{AC}}{\Delta\tau_{BC}} = 1.13^{+0.18}_{-0.17}$$

Barkana et al.1998

3.4: Weak Lensing





axies

ewhere

rce

Coherent distortion(shear) pattern of background galaxies

It shows Dark Matter distribution in Lensing Object

Lens equation $\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})$

Lens mapping $\delta\beta_i = A_{ij}(\vec{\theta}_0)\delta\theta_j$

$$A(\vec{\theta}_0)_{ij} = \delta_{ij} - \partial_i \partial_j \psi$$

$$A(\vec{\theta}_0) = (1-\kappa)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & -\gamma_1 \end{pmatrix} = (1-\kappa)\begin{pmatrix} 1-g_1 & -g_2 \\ -g_2 & 1+g_1 \end{pmatrix}$$

$$g_i = \frac{\gamma_i}{1-\kappa}, \quad i = 1,2$$

$$\kappa = \frac{1}{2}\Delta_\theta \psi = \frac{1}{2}(\psi_{,11} + \psi_{,22}),$$

$$\gamma_1 = \frac{1}{2}(\psi_{,11} - \psi_{,22}) = |\gamma|\cos(2\phi),$$

$$\gamma_2 = \psi_{,12} = |\gamma|\sin(2\phi)$$

Transformation of shear under the rotation

$$\vec{\partial} = \left(\frac{\partial}{\partial \theta_1}, \frac{\partial}{\partial \theta_2} \right) = (\partial_1, \partial_2) \rightarrow \vec{\partial}' = \left(\frac{\partial}{\partial \theta_1'}, \frac{\partial}{\partial \theta_2'} \right) = (\partial_1', \partial_2')$$

$$O = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}$$

$$\partial_1' = \cos\phi\, \partial_1 - \sin\phi\, \partial_2$$

$$\partial_2' = \sin\phi\, \partial_1 + \cos\phi\, \partial_2$$

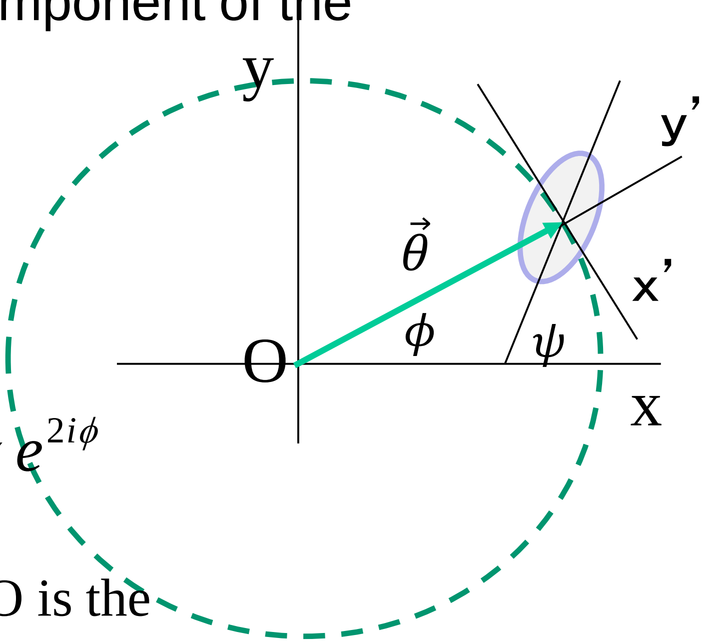
$$\gamma_1' = \frac{1}{2}\left(\partial_1'^2 - \partial_2'^2\right)\psi = \frac{1}{2}\left(\cos^2\phi - \sin^2\phi\right)\left(\partial_1^2 - \partial_2^2\right)\psi - \frac{1}{2}\sin\phi\cos\phi\, \partial_1 \partial_2 \psi$$

$$\gamma_2' = \partial_1' \partial_2' \psi = \sin\phi\cos\phi\left(\partial_1^2 - \partial_2^2\right)\psi + \left(\cos^2\phi - \sin^2\phi\right)\partial_1 \partial_2 \psi$$

$$\gamma' = \gamma_1' + i\gamma_2' = (\cos 2\phi + i \sin 2\phi)(\gamma_1 + i\gamma_2) = e^{2i\phi}\gamma$$

Tangential shear

Wen the surface mass distribution is nearly circular symmetric, It is convenient to deal with the tangential component of the shear w.r.t the center O



Shear w,r,t the x-y coordinate

$$\gamma = |\gamma| e^{2i\psi}$$

Shear w,r,t x'-y' the coordinate

$$\gamma' = |\gamma'| e^{2i(\pi/2+\psi-\phi)} = |\gamma| e^{2i(\pi/2+\psi-\phi)} = -\gamma\, e^{2i\phi}$$

Tangential component of shear w.r.t the center O is the x' component of the shear

$$\gamma_t(\vec{\theta}) = -Re(\gamma e^{-2i\phi}) = -\gamma_1 \cos 2\phi - \gamma_2 \sin 2\phi$$

Imaginary part of the shear w.r.t the x'-y' coordinate is called cross shear

$$\gamma_\times(\vec{\theta}) = -Im(\gamma e^{-2i\phi}) = \gamma_1 \sin 2\phi - \gamma_2 \cos 2\phi$$

Important relation between tangential shear and convergence

$$\langle \gamma_t(\theta) \rangle = \bar{\kappa}(\theta) - \langle \kappa(\theta) \rangle$$

where

$$\langle f(\theta) \rangle = \frac{1}{2\pi} \oint d\phi f(\theta,\phi) :$$ circumference average with radius θ

$$\bar{f}(\theta) = \frac{1}{\pi\theta^2} \int_0^\theta d\theta\theta \int_0^{2\pi} d\phi \ f(\theta,\phi) :$$ Average over a circle with radius θ

Proof

$$\int_0^\theta d^2\theta' \nabla \cdot \nabla\psi = \theta \oint d\phi \hat{n} \cdot \nabla\psi = \theta \oint d\phi \frac{\partial\psi}{\partial\theta}$$

$$l.h.s = 2\pi\theta^2 \bar{\kappa} = 4\pi \int_0^\theta d\theta'\theta' \langle \kappa(\theta') \rangle$$

$$4\pi \int_0^\theta d\theta'\theta' \langle \kappa(\theta') \rangle = \theta \oint d\phi \frac{\partial\psi}{\partial\theta}$$

Differentiate w.r.t θ

$$4\pi\theta \langle \kappa(\theta) \rangle = 2\pi\theta\bar{\kappa} + \theta \oint d\phi \frac{\partial^2\psi}{\partial\theta^2}$$

Polar coordinate expression for convergence and tangential shear

$$\kappa = \frac{1}{2}\left[\frac{\partial^2}{\partial\theta^2} + \frac{1}{\theta}\frac{\partial}{\partial\theta} + \frac{1}{\theta^2}\frac{\partial^2}{\partial\phi^2}\right]\psi$$

$$\gamma_t = -\frac{1}{2}\left[\frac{\partial^2}{\partial\theta^2} - \frac{1}{\theta}\frac{\partial}{\partial\theta} - \frac{1}{\theta^2}\frac{\partial^2}{\partial\phi^2}\right]\psi$$

$$\Longrightarrow \frac{\partial^2\psi}{\partial\theta^2} = \kappa - \gamma_t$$

$$4\pi\theta\langle\kappa\rangle = 2\pi\theta\,\bar{\kappa} + \theta\oint d\phi(\kappa - \gamma_t) = 2\pi\theta\bar{\kappa} + 2\pi\theta(\langle\kappa\rangle - \langle\gamma_t\rangle)$$

$$\Longrightarrow \langle\gamma_t(\theta)\rangle = \bar{\kappa}(\theta) - \langle\kappa(\theta)\rangle$$

Example: NFW profile

$$<\gamma_t>_{\rm NFW}(\theta) = \kappa_s\left(\frac{2g(x)}{x^2} - f(x)\right)$$

Thus the average of tangential shear over circumference with radius θ gives us the information of surface mass density inside the raius θ

$$\Sigma_+ \equiv \Sigma_{crit}\langle\gamma_t(\theta)\rangle$$ Differential surface density

Kaiser-Squire Mass Reconstruction

$\gamma(\vec{\theta})$ → $\kappa(\vec{\theta})$

Observable → Surface mass density

$$\hat{\gamma}(k) = \frac{1}{\pi}\hat{\kappa}(k)\hat{D}(k)$$

$$\hat{D}(k) = \pi \frac{k_1^2 - k_2^2 - 2ik_1k_2}{k^2} = \pi\ e^{2i\phi},\ \ \vec{k} = (k,\phi)$$

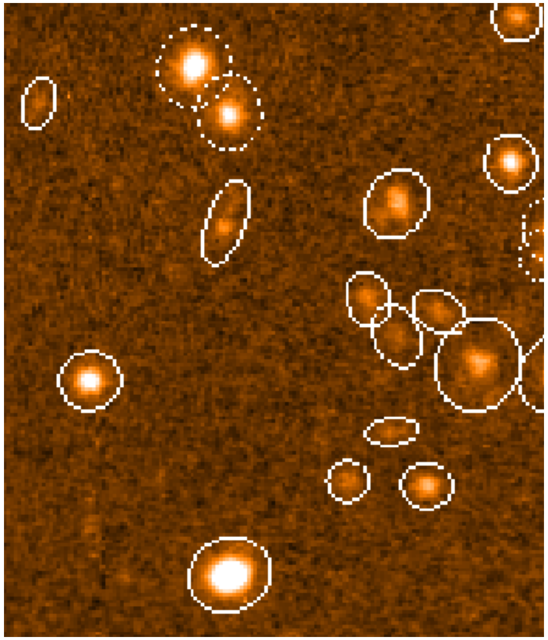
$$\hat{\kappa}(k) = \frac{1}{\pi}\hat{\gamma}(k)\hat{D}^*(k)$$

How to measure the gravitational shear

1) Measures the 2nd-moments of the surface brightness f(θ) of individual galaxies

Define the center of the image

$$\bar{\theta}_i = \frac{\int d^2\theta\, q[I(\vec{\theta})]\theta_i}{\int d^2\theta\, q[I(\vec{\theta})]}$$



$q[I(\vec{\theta})]$: weighting function of surface brightness I

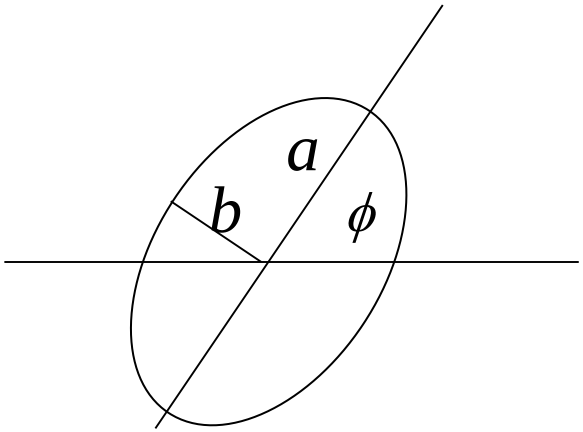
$$Q_{ij}^{(obs)} = \frac{\int d^2\theta\, q[f(\vec{\theta})](\theta-\bar{\theta})_i(\theta-\bar{\theta})_j}{\int d^2\theta\, q[f(\vec{\theta})]}$$

In the actual observation we need a window function and PSF correction

2) Defines the components of ellipticities of a galaxy image

$$\vec{e}^{\,obs} = \left(e_1^{obs}, e_2^{obs}\right) = \left(\frac{Q_{11}^{obs} - Q_{22}^{obs}}{Q_{11}^{obs} + Q_{22}^{obs}} \quad \frac{2Q_{12}^{obs}}{Q_{11}^{obs} + Q_{22}^{obs}} \right)$$

$$e^{obs} := e_1 + ie_2 = \frac{a^2 - b^2}{a^2 + b^2} e^{2i\phi}$$



3) Relates intrinsic and observed ellipticities

$$Q_{ij}^{(s)} = \frac{\int d^2\beta \; q[I^{(s)}(\vec{\beta})](\beta - \bar{\beta})_i (\beta - \bar{\beta})_j}{\int d^2\beta \; q[f^{(s)}(\vec{\beta})]}$$

$$(\ \beta = \theta - \partial\psi \rightarrow \delta\beta = A(\theta)\delta\theta\ )$$

$$= \frac{\int d^2\theta \det A(\theta) \; q[I(\vec{\theta})] A_{ik}(\bar{\theta})(\theta - \bar{\theta})_k A_{j\ell}(\bar{\theta})(\theta - \bar{\theta})_\ell}{\int d^2\theta \det A(\theta) \; q[I(\vec{\theta})]} = A_{ik}(\bar{\theta}) Q_{k\ell}^{obs} A_{j\ell}(\bar{\theta})$$

Conservation of surface brightness

$$I^{(s)}(\delta\beta) = I(A\delta\theta) \equiv I(\theta)$$

$$e^{(s)} = \frac{e - 2g + g^2 e^*}{1 + |g|^2 - 2\,\mathrm{Re}(g e^*)}$$

$$g \equiv \frac{\gamma}{1-\kappa}:$$ Reduced shear

In the weak lensing limit

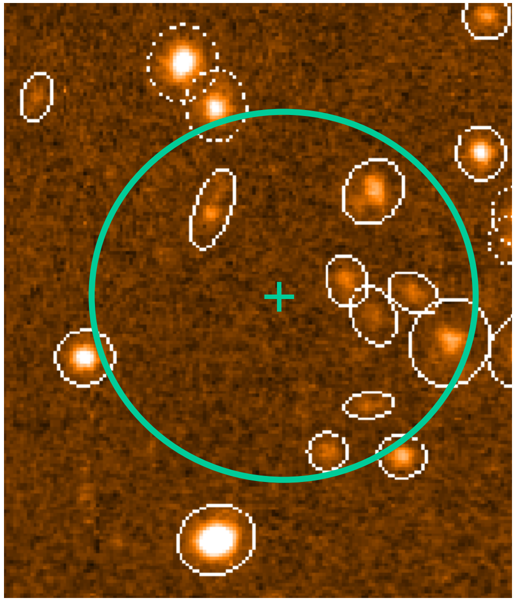
$$\kappa,\ |\gamma| << 1,\ \ g \approx \gamma$$

$$e^{obs} \approx e^{(s)} + 2\gamma$$

4) Assuming the random orientations of intrinsic galaxy ellipticities:

$$< e^{(s)} > (\vec{\theta}) \approx 0$$

$$< e^{(obs)} > (\vec{\theta}) = 2 < \boldsymbol{\gamma} > (\vec{\theta}) + O\left(\frac{\sigma_{\rm int}}{\sqrt{N}} \right)$$



Averaging

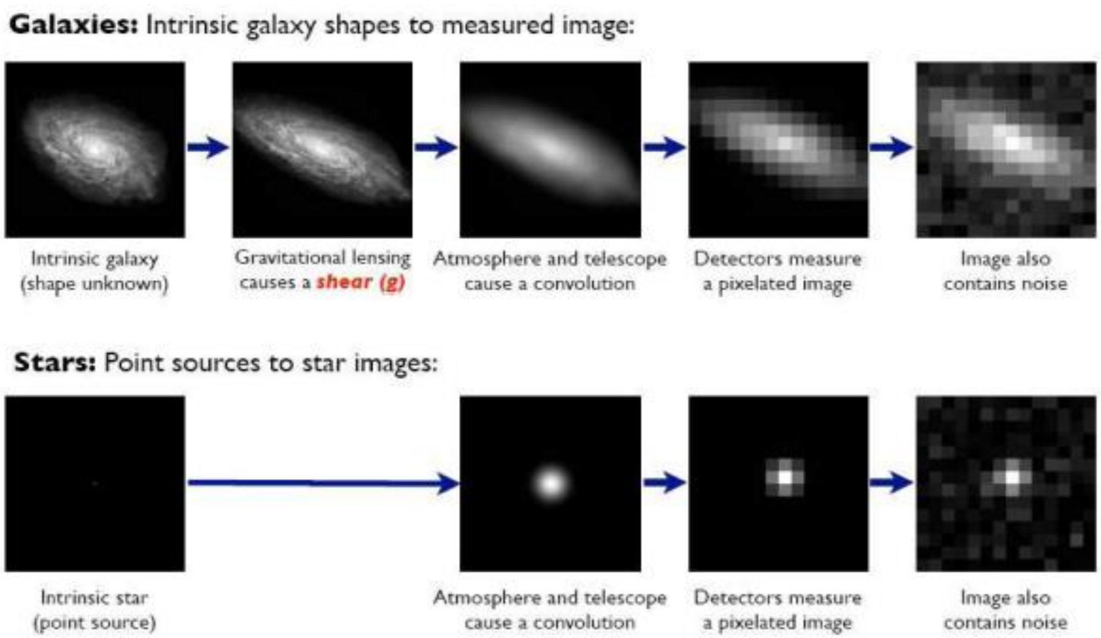
$$< e^{(obs)} > (\vec{\theta}) \equiv \frac{\sum_i u(|\vec{\theta} - \vec{\theta}_i|) e(\vec{\theta}_i)}{\sum_i u(|\vec{\theta} - \vec{\theta}_i|)}$$

$\sigma_{\rm int} \sim 0.2 - 0.3$: stand. deviation of the intrinsic ellipticities

$N \approx 20 - 30$: averaged number of galaxies

Observable galaxy ellipticities are unbiased estimator of the gravitational shear

In reality we need PSF correction



Bridle et al.2008

$$I^{OBS}(\vec{\theta}) = \int d^2\theta' I^{(obs)}(\vec{\theta}')P(\vec{\theta}-\vec{\theta}')$$

Point Spread Function(PSF)

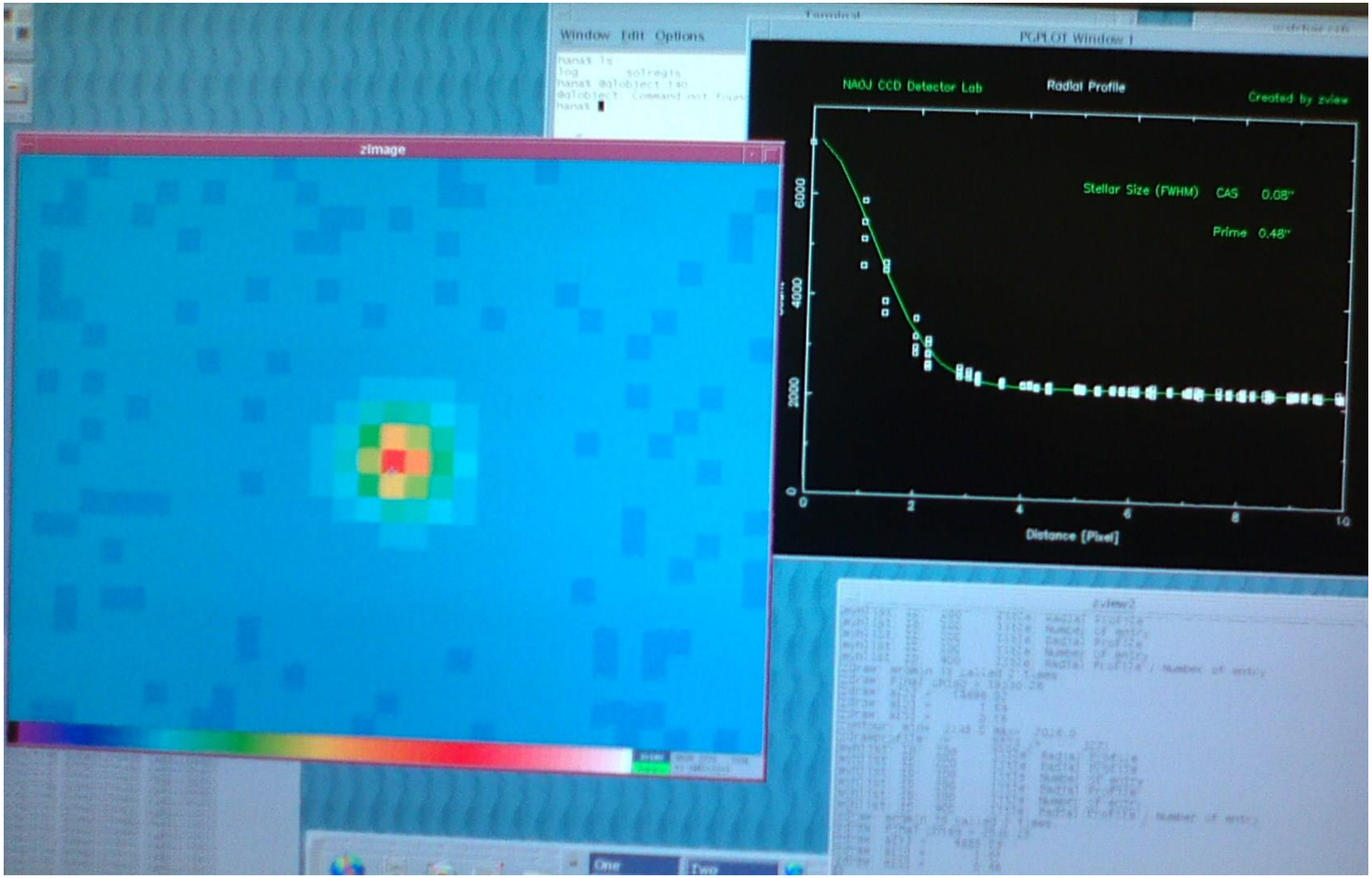
time dependent in ground base telescope

For stars

$$I^{star}(\vec{\theta}_{star}) = \int d^2\theta' \delta(\vec{\theta}')P(\vec{\theta}_{star}-\vec{\theta}') = P(\vec{\theta}_{star})$$

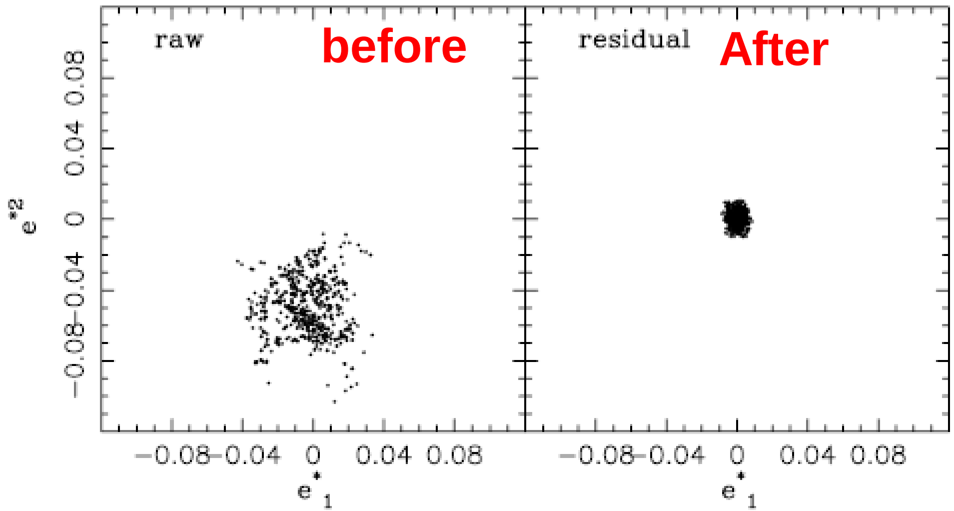
Number density of star $n_{star} \approx 1 \text{ arcmin}^{-2}$

Star image in a good night with 0.48”



Result of PSF correction

Stellar ellipticity

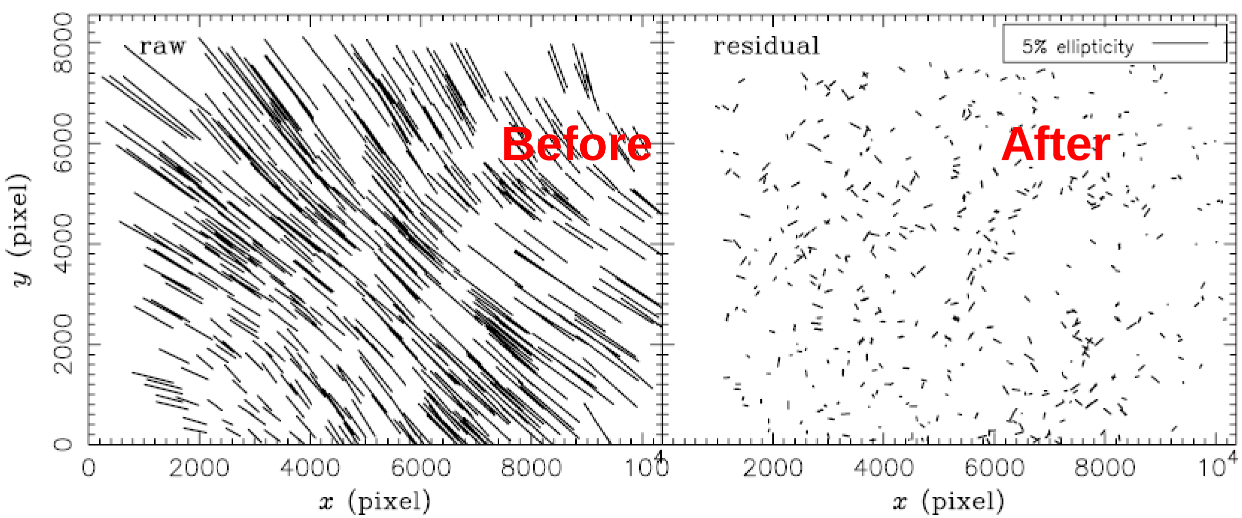


Averaged value before correction

$$e_1^{star} \cong -3.5\times10^{-3}, \quad e_1^{star} \cong -5.0\times10^{-2}$$

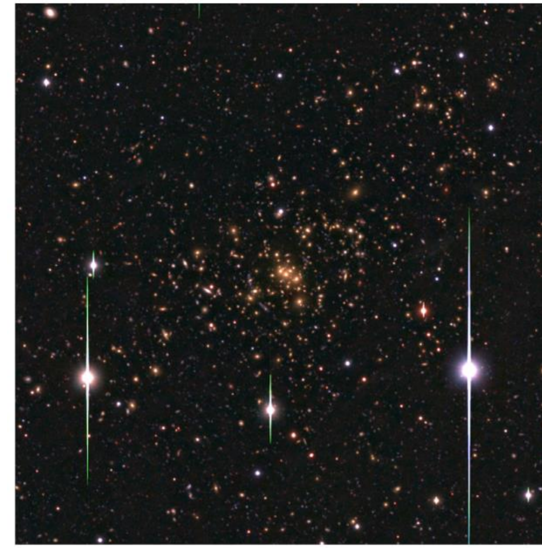
After the correction

$$e_1^{star} = (-0.07 \pm 1.18)\times10^{-4},$$
$$e_1^{star} \cong (2.10 \pm 1.58)\times10^{-4}$$



Importance of selection of background galaxies

例: Cl 0024+1654(Umetsu et.al. 2010)



Cl 0024+1654(z=0.395)

Dilution effect

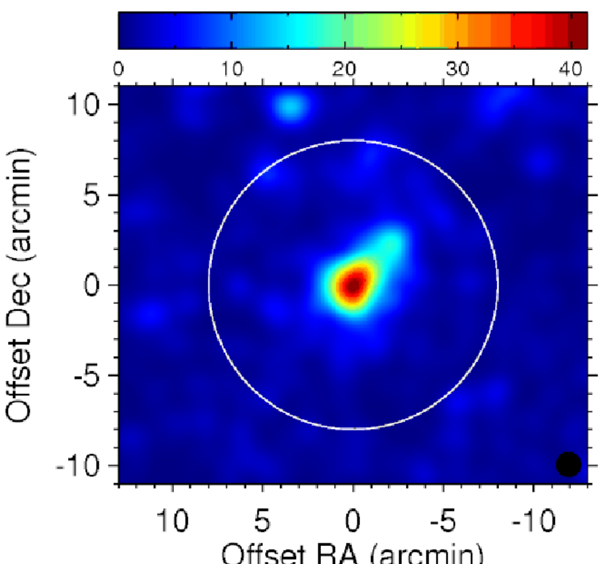
If there is a contamination from background galaxies

Averaged shear signal: $g' = g_{true} \cdot N_{bg} / N_{tot} < g_{true} \; : \; N_{tot} = N_{cl} + N_{bg}$

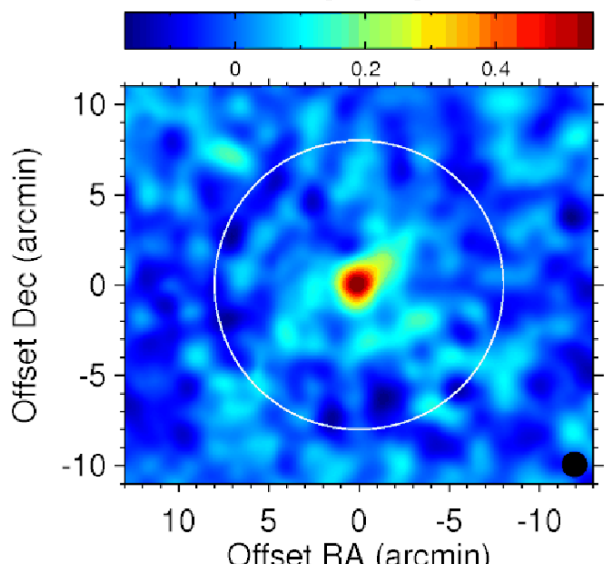
noise $n' = \sigma_g / \sqrt{N_{tot}} = n_{true} \sqrt{N_{bg} / N_{tot}} < n_{true}$

$$\Rightarrow \quad \left(\frac{S}{N}\right)^{obs} = \left(\frac{S}{N}\right)^{true} \sqrt{\frac{N_{bg}}{N_{tot}}} = \left(\frac{S}{N}\right)^{true} \frac{1}{\sqrt{1+f}}; \qquad f \equiv \frac{N_{cl}}{N_{bg}}$$

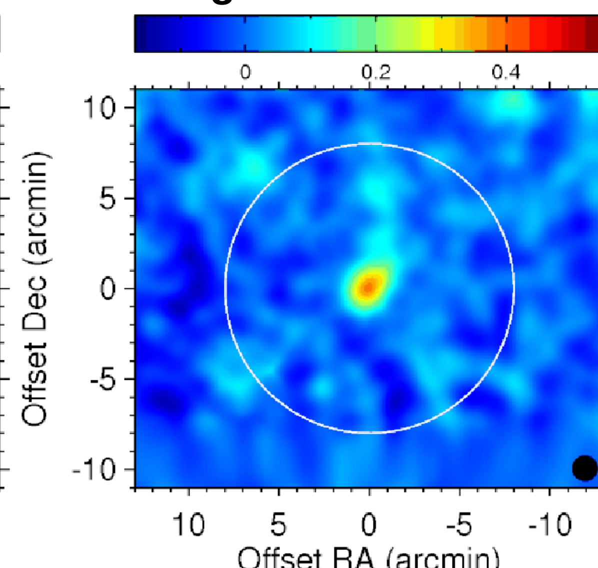
Galaxy Number density



Color-color(BRz) selected



Magnitude selected



Mass sheet degeneracy and magnification bias

Weak lensing observable is reduced shear

$$g(\vec{\theta}) = \frac{\gamma(\vec{\theta})}{1-\kappa(\vec{\theta})}$$

This field is invariant under the following global transformation

$$\kappa(\vec{\theta}) \to \lambda\kappa(\vec{\theta}) + 1 - \lambda,\ \ \gamma(\vec{\theta}) \to \lambda\gamma(\vec{\theta})$$

It corresponds to the uniform density sheet

Mass reconstruction using only distortion cannot break this degeneracy

However magnification changes by this transformation

$$\mu(\boldsymbol{\theta}) = \frac{1}{[1-\kappa(\boldsymbol{\theta})]^2 - |\gamma(\boldsymbol{\theta})|^2},\ \ \to \lambda^{-2}\mu$$

There are two effects by lens magnification

$$\delta\Omega^{\rm obs} = \mu(\vec{\theta})\delta\Omega^S$$ Expansion of area in sky

$$S^{\rm obs} = \mu(\vec{\theta})S^S$$ Enfacement of the observed flux

The nnlensed number count per solid angle

$$n_0(>S_0) \equiv \int_{S_0}^{\infty} dS \frac{d^2N}{d\Omega dS} \propto S_0^{-\alpha} \qquad \alpha \equiv -\frac{d\log_{10} n_0(>S_0)}{d\log_{10} S_0}$$

lens

$$n(>S_0) = \int_{S_0/\mu}^{\infty} dS \frac{d^2N}{\mu d\Omega dS} = \mu^{\alpha-1} n_0(>S_0)$$

In the week lensing limit,

$$n(>S_0) \simeq (1+2\kappa)^{\alpha-1} n_0(>S_0) \simeq (1+2(\alpha-1)\kappa)\, n_0(>S_0)$$

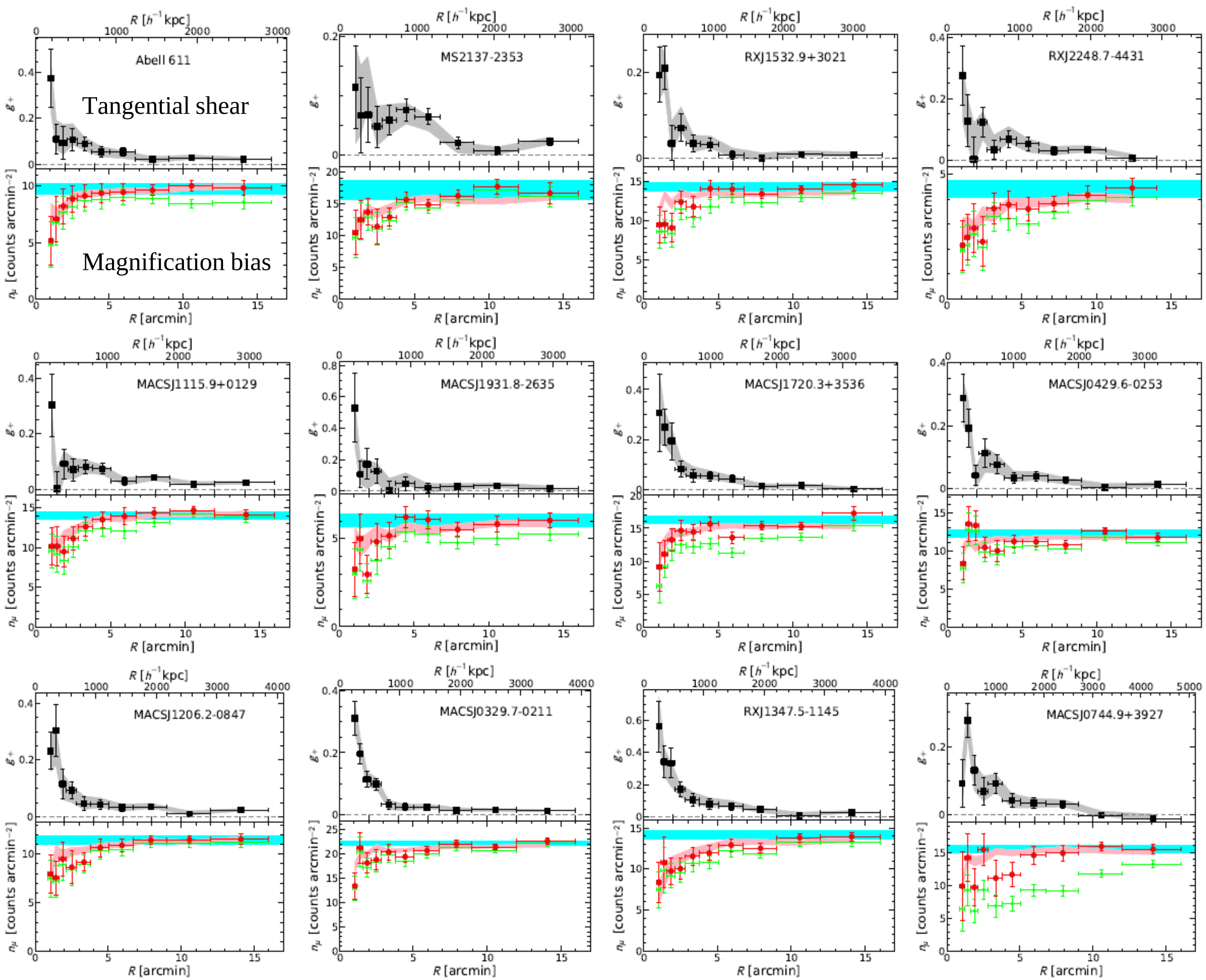
The fractional change in the number density of background objects

$$\delta_N = \frac{\delta n}{n_0} \simeq -2(1-\alpha)\kappa$$

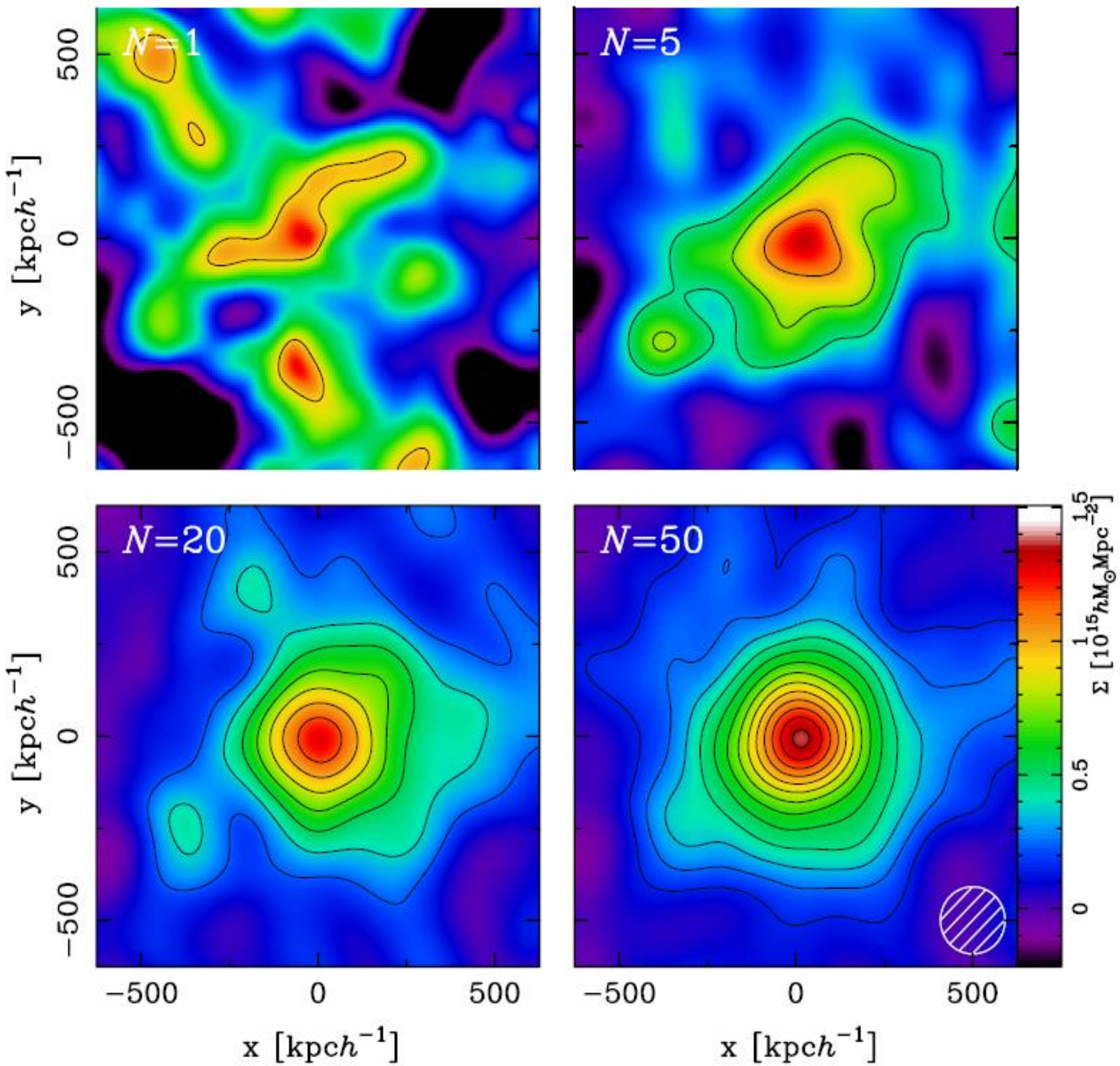
Observation of shear and magnification, Umetsu et al, 2014

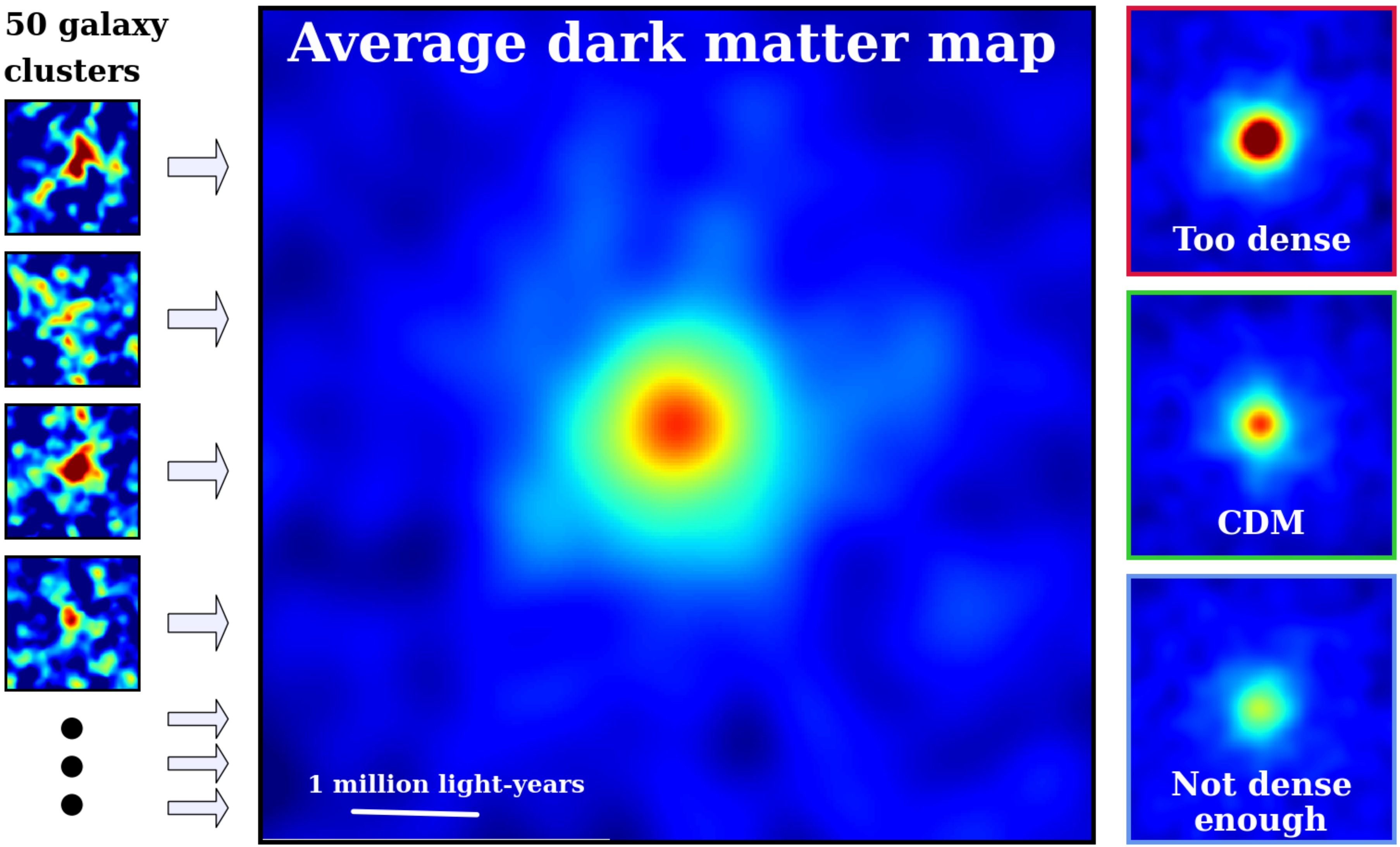
CLASH(Cluster Lensing And Supernova survey with Hubble)

25 clusters at $0.18 < z < 0.89$,



Weak Lensing Analysis for 50 clusters (0.15 <z<0.30) with Subaru




50 galaxy clusters
Average dark matter map
1 million light-years
Too dense
CDM
Not dense enough

NFW profile

A phenomenological model for DM halos motivated by simulation

$$\rho_{\rm NFW}(r) = \frac{\rho_{\rm s}}{(r/r_{\rm s})(1 + r/r_{\rm s})^2},$$

ρ_s and r_s are the characteristic density and radius

$$\frac{d\log\rho}{d\log r} = -2 \quad \text{at } r = r_s$$

NFW profile is also characterized by the following two parameters

$$M_{vir} = \int_0^{r_{vir}} dr\, 4\pi r^2 \rho(r) = 4\pi\rho_s r_s^3 \left[\ln(1+c) - \frac{c}{1+c}\right] \qquad c \equiv \frac{r_{vir}}{r_s}$$

$$\equiv \frac{4\pi}{3}\rho_s\ r_{vir}^3$$

Surface mass density

$$\Sigma(R) = 2\rho_s r_s F(X) \qquad X \equiv R / r_s$$

where

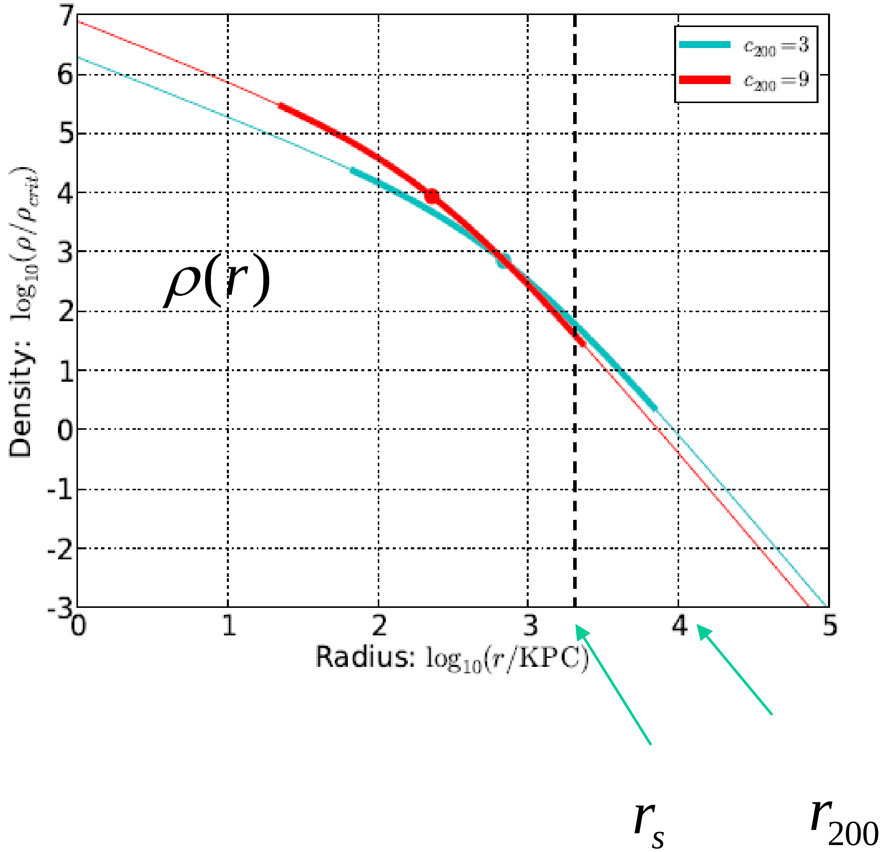
$$F(X) = \begin{cases} \frac{1}{X^2-1}\left(1 - \frac{1}{\sqrt{1-X^2}}\cosh^{-1}\frac{1}{X}\right) & (X<1) \\ \frac{1}{3} & (X=1) \\ \frac{1}{X^2-1}\left(1 - \frac{1}{\sqrt{X^2-1}}\cos^{-1}\frac{1}{X}\right) & (X>1) \end{cases}$$

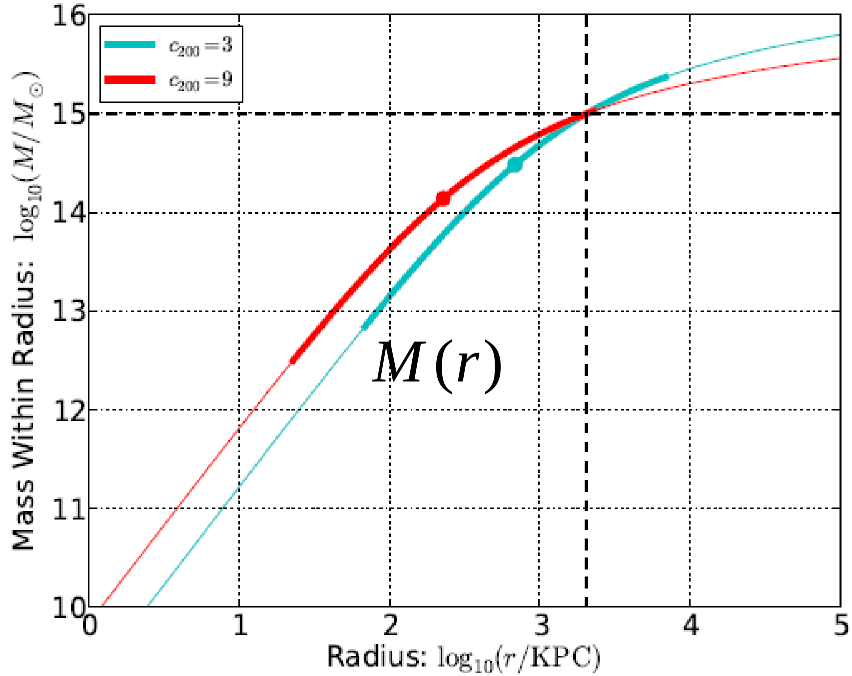
2D projected mass

$$M(R) = 4\pi r_s^3 \rho_s G(X)$$

$$G(X) = \ln\frac{X}{2} + \begin{cases} \frac{1}{\sqrt{1-X^2}}\cosh^{-1}\frac{1}{X} & (X<1) \\ 1 & (X=1) \\ \frac{1}{\sqrt{X^2-1}}\cos^{-1}\frac{1}{X} & (X>1) \end{cases}$$

NFW profile with c=3, 9 for $M_{200} = 10^{15} M_{sun}$





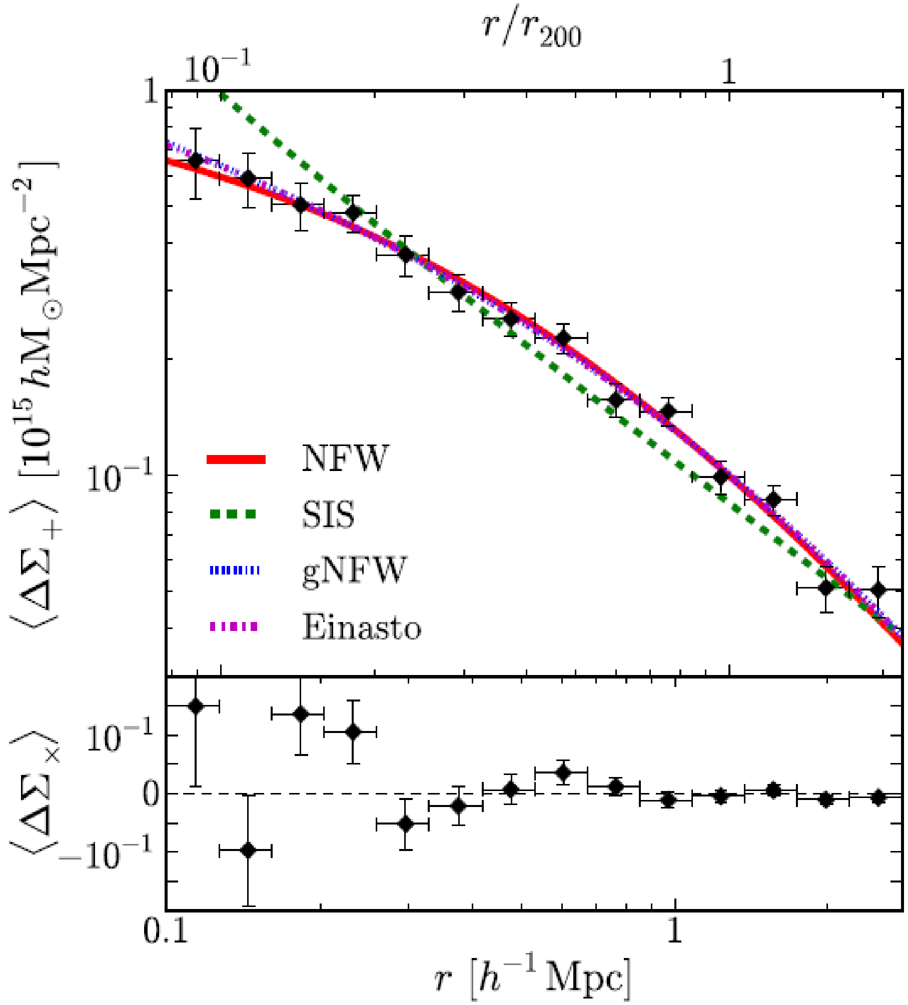
- SIS(singular Isothermal Sphere)

$$\rho_{\rm SIS}(r) = \frac{\sigma_v^2}{2\pi G r^2}$$

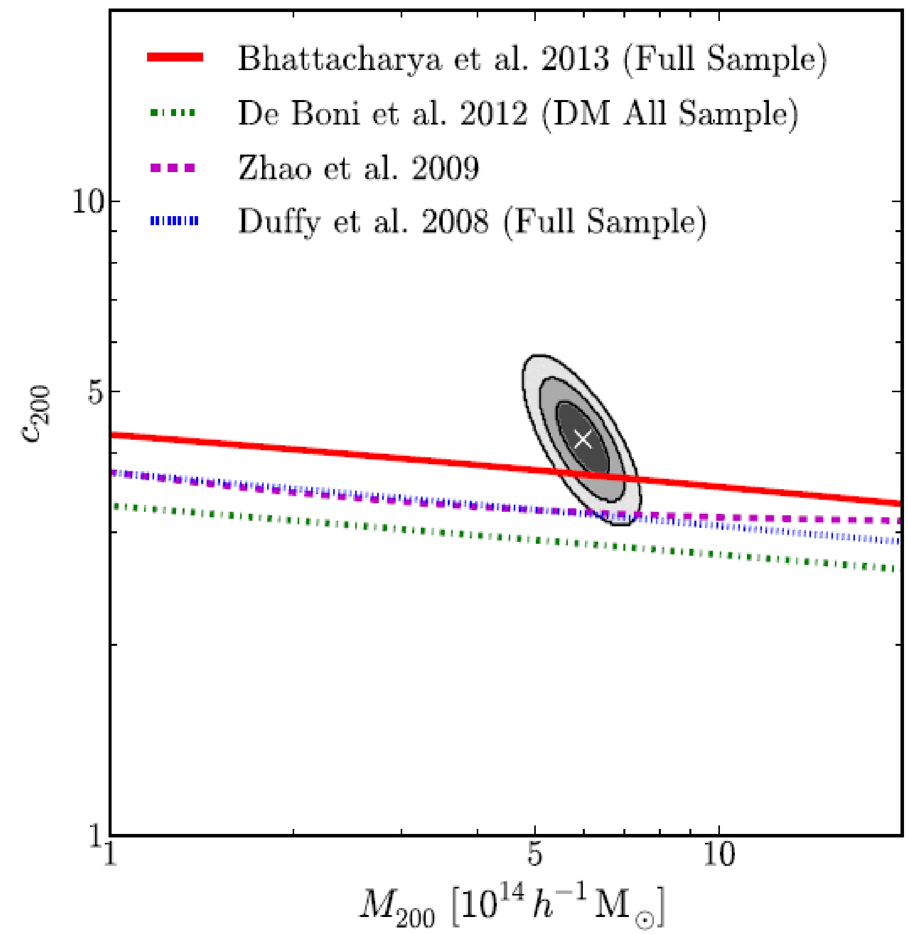
- Einasto profile

$$\rho_{\rm E}(r) = \rho_{\rm s} \exp\left[-\frac{2}{\alpha_{\rm E}}\left(\frac{r}{r_{\rm s}}\right)^{\alpha_{\rm E}}\right]$$

with $\alpha_{\rm E}$ the shape parameter describing the degree of curvature. An Einasto profile with $\alpha_{\rm E} \approx 0.18$ closely resembles the NFW profile over roughly two decades in radius (Ludlow et al. 2013). The logarithmic density gradient equals -2 at $r = r_{\rm s}$.

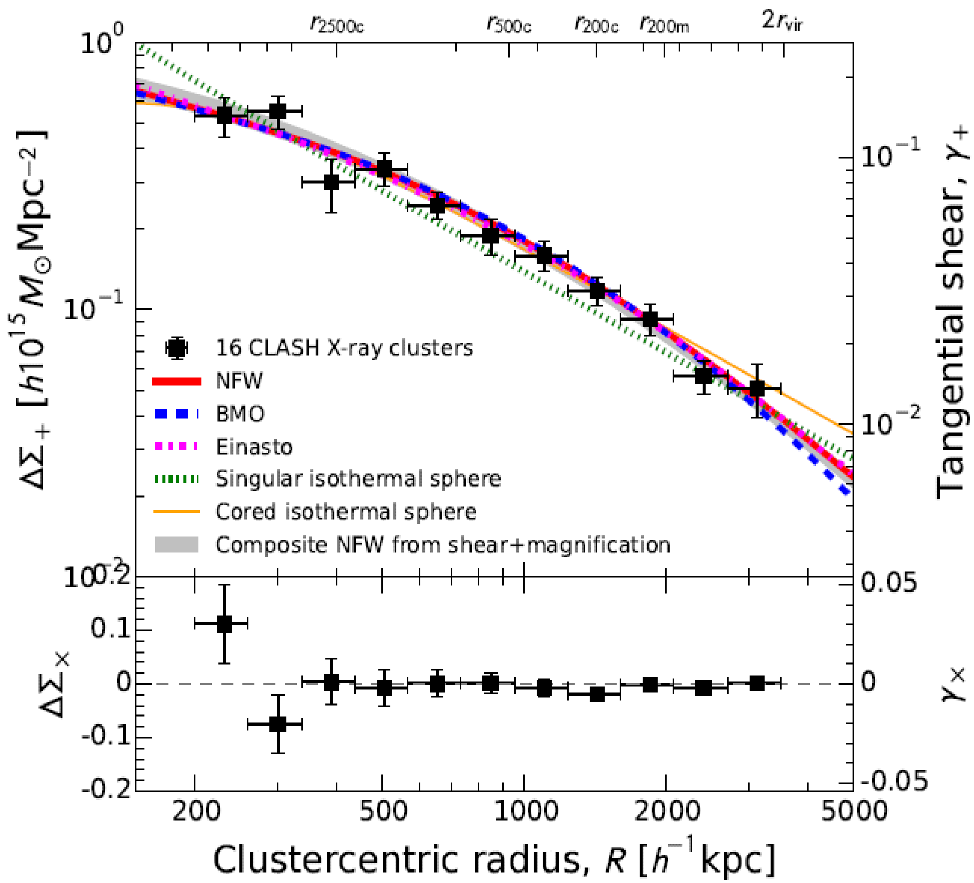


The averaged tangential shear profile obtaind from stacking 50 clusters with <z>-0.23

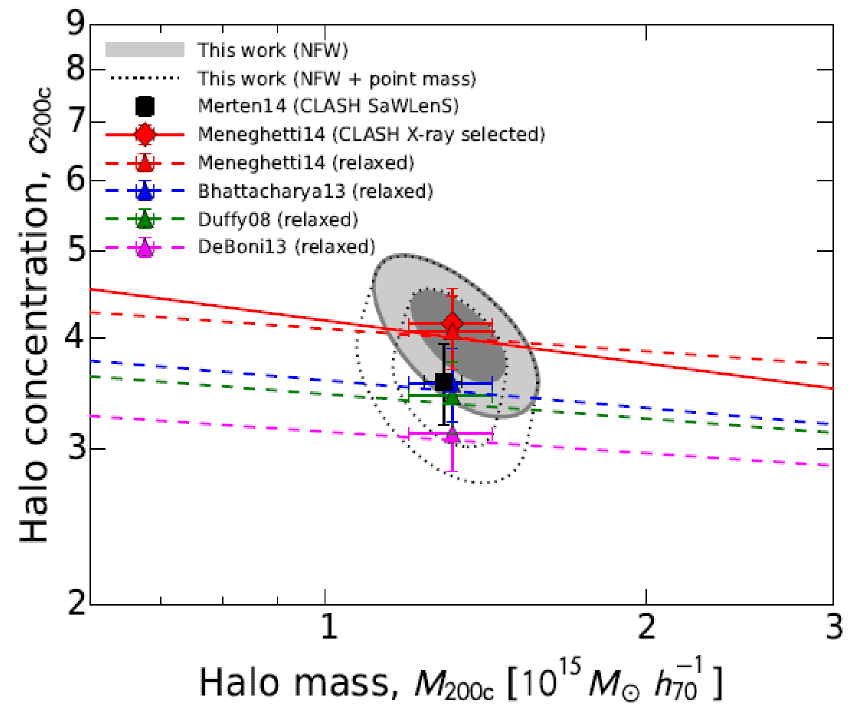


Mass and concentration of 50 clusters at <z_l>=0.23

CLASH sample



The averaged tangential shear profile obtaind from stacking the X-ray selected subsample of 16 clusters with <z_l>=0.35



Mass and concentration of 16 CLASH X-ray selected clusters at <z_l>=0.35

Weak Lensing by LSS

- Cosmic shear

  Dark energy

- Distance-redshift relation

Cosmic shear

$$\langle \gamma(\theta_1)\gamma(\theta_2)\rangle \rightarrow P_\gamma(\ell)_{obs} \Leftrightarrow P_\gamma(\ell)_{theory}$$

Light propagation in an inhomogeneous universe

$$\vec{\alpha}(\lambda) = \frac{2}{c^2}\int_0^\lambda d\lambda' \frac{r(\lambda' - \lambda_s)}{r(\lambda_s)} \vec{\nabla}_\perp \Phi$$

Gravi. potential of LSS

Convergence(mass)

$$\kappa(\vec{\theta},\lambda) = \frac{1}{2}\vec{\nabla}_\theta \cdot \vec{\alpha}(\vec{\theta}) = \frac{1}{c^2}\int_0^\lambda d\lambda' \frac{r(\lambda' - \lambda_s)r(\lambda')}{r(\lambda_s)} \Delta\Phi(r(\lambda')\vec{\theta},\lambda')$$

Poisson equation $\Delta\Phi = 4\pi G a^2 \rho_0 \delta = \dfrac{3H_0^2\Omega_{m0}}{2a}\delta$ ← Density fluctuation

$$\kappa(\vec{\theta}) = \int_0^{\lambda_H} d\lambda\, p(\lambda)\kappa(\lambda,\vec{\theta}) = \frac{3H_0^2\Omega_{m0}}{2c^2}\int_0^{\lambda_H} d\lambda \frac{\lambda\, g(\lambda)}{a(\lambda)}\delta(\lambda,\vec{\theta})$$

Redshift distribution
of source galaxies

$$g(\lambda) = \int_0^{\lambda_H} d\lambda' p(\lambda')\frac{D_{ls}}{D_l}$$

Two point statistics

$$\left\langle \kappa(\vec{\theta})\kappa(\vec{\theta}')\right\rangle = \int \frac{d^2k}{(2\pi)^2}\int \frac{d^2k'}{(2\pi)^2} \left\langle \kappa(\vec{k})\kappa(\vec{k}')\right\rangle e^{-i\vec{k}\cdot\vec{\theta}} e^{-i\vec{k}'\cdot\vec{\theta}'}$$

Power spectrum

$$\left\langle \kappa(\vec{k})\kappa^*(\vec{k}')\right\rangle \equiv (2\pi)^2 \delta(\vec{k}-\vec{k}')P_\kappa(k)$$

Power spectrum for κ is expressed by matter power spectrum

$$P_\kappa(k) = \frac{9H_0^2\Omega_{m0}^2}{4}\int_0^{\chi_H} d\chi \frac{g^2(\chi)}{a^2(\chi)} P_\delta\left(\frac{k}{\chi};\chi\right)$$

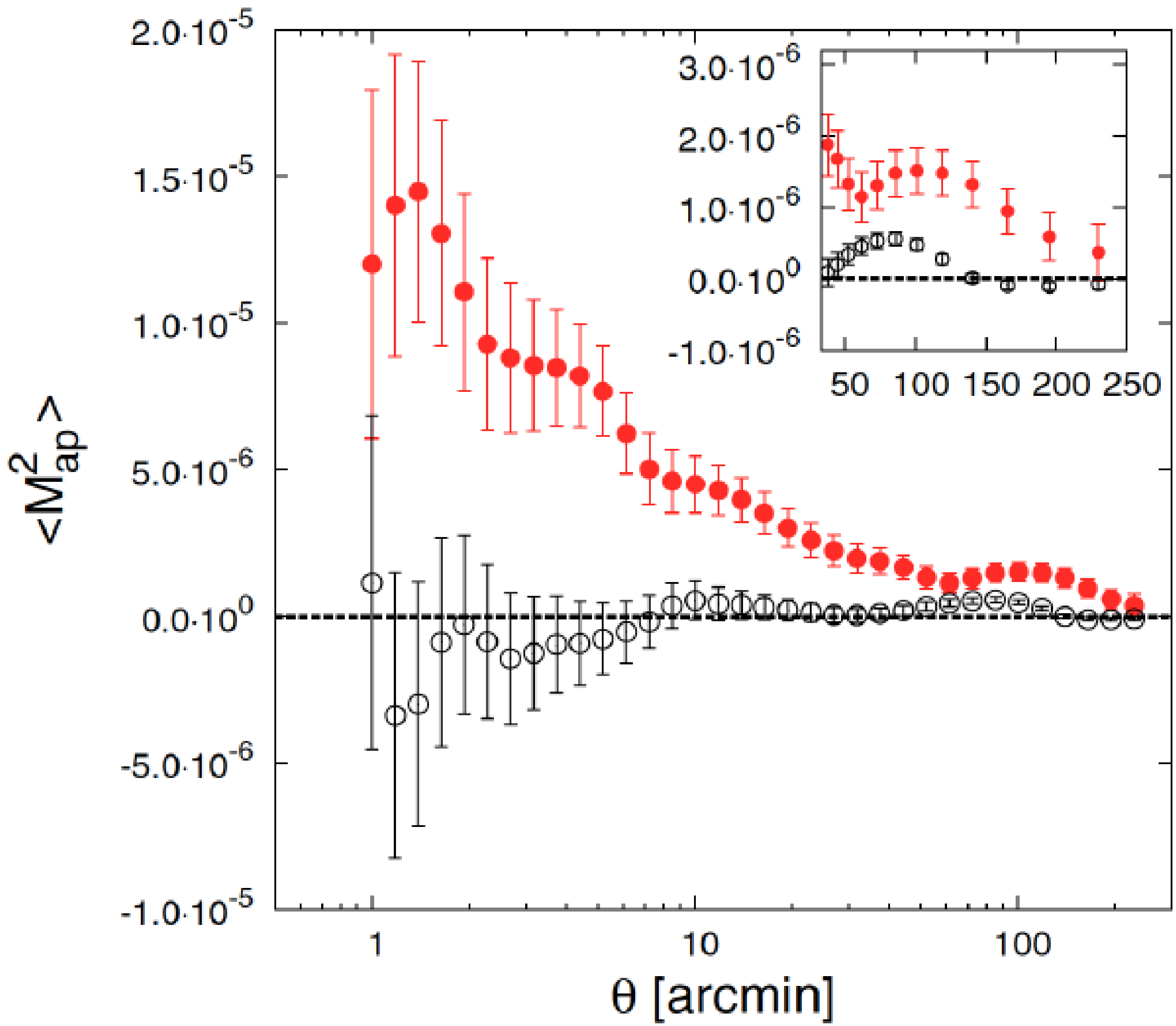
$$\left\langle \kappa(\vec{\theta})\kappa(\vec{\theta}')\right\rangle = \int \frac{d^2k}{(2\pi)^2} P_\kappa(k)e^{-i\vec{k}\cdot\vec{\theta}} = \int \frac{kdk}{2\pi} P_\kappa(k)J_0(k\theta)$$

$$\gamma(k) = \frac{k_1^2 - k_2^2 + ik_1k_2}{k^2}\kappa(k) \Rightarrow \left\langle \gamma(\vec{\theta})\gamma(\vec{\theta}')\right\rangle = \left\langle \kappa(\vec{\theta})\kappa(\vec{\theta}')\right\rangle$$

$$\Downarrow$$

$$P_\kappa(\ell)_{obs} \Leftrightarrow P_\kappa(\ell;\omega_{DE})_{theor}$$

CFHT result (Fu et.al 2008)



Difficulties

- Accurate shape measurement
- Accurate photo-z
- Accurate PSF correction